$\mathcal{A}$ - SETS IN BIMINIMAL STRUCTURE SPACES

BY

CHANIKA KULKHOR

A thesis submitted in partial fulfillment of the requirements for
the degree of Master of Science in Mathematics
at Mahasarakham University
June 2018

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Examining Committee

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(Asst. Prof. Supunnee Sompong, Ph.D.)	(External expert)
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(Prof. Wichian Magtoon, Ph.D.)	(Asst Prof. Krit Chaimoon, Ph.D.)
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ชื่อเรื่อง เซต $\mathcal{A}$ ในปริภูมิสองโครงสร้างเล็กสุด

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บทคัดย่อ

งานวิจัยนี้ผู้วิจัยได้นำเสนอแนวคิดของเซต $\mathcal{A}$ ในปริภูมิสองโครงสร้างเล็กสุด และศึกษาสมบัติของเซต $\mathcal{A}$ ในปริภูมิสองโครงสร้างเล็กสุด นอกจากนี้ยังศึกษาสมบัติของฟังก์ชันต่อเนื่อง $\mathcal{A}$, ศึกษาสมบัติของเซตแยกกันและเซตเชื่อมโยง $\mathcal{A}$ ในปริภูมิสองโครงสร้างเล็กสุด

คำสำคัญ : เซต $\mathcal{A}$, ฟังก์ชันต่อเนื่อง $\mathcal{A}$, เซตแยกกันและเซตเชื่อมโยง $\mathcal{A}$

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ABSTRACT

In this research, we introduce the concepts of $\mathcal{A}-$ sets in biminimal structure spaces and investigate some of their properties. Moreover, the notion $\mathcal{A}-$ sets, $\mathcal{A}-$ continuous functions, $\mathcal{A}-$separated sets and $\mathcal{A}-$connected sets in biminimal structure spaces were studied.

Keywords : $\mathcal{A}-$ set, $\mathcal{A}-$ continuous function, $\mathcal{A}-$separated set and $\mathcal{A}-$connected set.

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Background

In 1972, J. Dugundji [7] introduced the concepts of regular closed sets in topological spaces. Let (X,τ) be a topological space and let $A \subseteq X$. Then A is called regular closed if and only if $A = Cl(Int(A))$. In 2003, Á.Császár [6] introduced the concepts of $\gamma-$connected sets in topological spaces. Also he studied $\gamma-$closed sets, $\gamma-$open sets and $\gamma-$separated sets. In 1986, J. Tong [21] introduced the concepts and properties of $\mathcal{A}-$sets in topological spaces. Let A be a subset of a topological space (X,τ), then A is an $\mathcal{A}-$ set in (X,τ) if there exist U and B, such that $A = U \cap B$ when U is open and B is regular closed in (X,τ). In addition, J. Tong [21] introduced the concepts of $\mathcal{A}-$ continuous functions from a topological space (X,τ) to a topological space $(Y,\mathcal{U})$. Let f be a function from X to Y, then f is $\mathcal{A}-$continuous function if and only if the inverse image of each open set in Y is an $\mathcal{A}-$set in X. In 1990, M. Ganster, and Reilly, I. L. [9] improved J. Tong's decomposition result and provide a decomposition of $\mathcal{A}-$ continuity. In 2000, the concepts of minimal structure spaces were introduced by V.Popa and T.Noiri [18]. A pair (X, m_X) is a minimal structure space if and only if $X \neq \emptyset$ and m_X is family of $P(X)$ with $\emptyset, X \in m_X$. Moreover, they also introduced the concepts of m_X-open sets and m_X-closed sets in minimal structure spaces. Other from this, such definitions were used to define m_X-interior and m_X- closure operators, respectively. In 2010, W. Keun Min [11] introduced the concepts of $\alpha m-$open sets, $\alpha-$interior and $\alpha m-$closed operators in minimal structure space. In 1963, J.C.Kelly [10] introduced the concepts of bitopological spaces which consist of an empty set and two topological spaces. In 2010, C.Boonpok [3] introduced the concepts of the spaces which consist of an empty set and two minimal structures is called biminimal structure spaces. Furthermore, this C.Boonpok [3] defined $m^1_X m^2_X-$ closed set in biminimal structure spaces and the complement of $m^1_X m^2_X-$ closed sets is call $m^1_X m^2_X-$ open sets. In 2010, C.Boonpok [4] defined $(i,j)m_X-$ regular open sets in biminimal structure spaces and he also defined $(i,j)m_X-$ regular closed sets as complement of $(i,j)m_X-$ regular open sets for $i,j = 1,2$ and $i \neq j$

The development of the research mentioned above. Researcher interested to define the study of some properties of $\mathcal{A}-$ sets and including $\mathcal{A}-$continuous functions in biminimal structure spaces.

CHAPTER 2

PRELIMINARIES

In this chapter, we will give some definitions, notations, dealing with some preliminaries and some useful results that will be duplicated in later chapter.

2.1 Topological spaces

This section, we recall some notions, notations and previous results.

Definition 2.1.1. [20] Let X be a nonempty set. A class τ of subsets of X is a $topology$ on X iff τ satisfies the following axioms:

(1) X and $\emptyset$ belong to τ;

(2) The union of any number of sets in τ belongs to τ;

(3) The intersection of any two sets in τ belongs to τ;

The elements of τ are then called $open\ sets$ and there complements are called $closed\ sets$, the pair (X,τ) is called a $topological\ space$.

Definition 2.1.2. [20] Let (X,τ) be a topological space and $A \subseteq X$. The $interior$ of A and the $closure$ of A are defined as follow:

(1) $Int(A) = \bigcup\{U : U \subseteq A, U \in \tau\}$;

(2) $Cl(A) = \bigcap\{F : A \subseteq F, X \setminus F \in \tau\}$.

Definition 2.1.3. [12] Let (X,τ) be a topological space and $A \subseteq X$. Then A is called $semi-open$ if and only if $A \subseteq Cl(Int(A))$.

The family of all semi$-$open sets in a topological spaces (X,τ) is denoted by $SO(X,\tau)$.

Definition 2.1.4. [12] Let (X,τ) be a topological space and $A \subseteq X$. Then A is called $semi-closed$ if and only if $X \setminus A$ is semi$-$open .

Definition 2.1.5. [13] Let (X,τ) be a topological space and $A \subseteq X$. Then A is called $pre-open$ if and only if $A \subseteq Int(Cl(A))$.

The family of all pre$-$open sets in a topological spaces (X,τ) is denoted by $PO(X,\tau)$.

Definition 2.1.6. [13] Let (X, τ) be a topological space and $A \subseteq X$. Then A is called $pre-closed$ if and only if $X \setminus A$ is pre$-$open.

Proposition 2.1.7. [1] Let (X, τ) be a topological space and $A \subseteq X$. Then A is pre$-$closed if and only if $Cl(Int(A)) \subseteq A$.

Definition 2.1.8. [8] Let (X, τ) be a topological space and $A \subseteq X$. The $pre-closure$ of a subset A, denoted by $pcl(A)$ is the intersection of all pre$-$closed subsets of (X, τ) that contain A .

Proposition 2.1.9. [1] Let (X, τ) be a topological space and $A \subseteq X$. Then $pcl(A) = A \cup Cl(Int(A))$.

Definition 2.1.10. [16] Let (X, τ) be a topological space and $A \subseteq X$. Then A is called an $\alpha-set$ if and only if $A \subseteq Int(Cl(Int(A)))$.

The family of all $\alpha-$ sets in a topological spaces (X, τ) is denoted by τ^{α}.

Definition 2.1.11. [1] Let (X, τ) be a topological space and $A \subseteq X$. Then A is called $semi-preopen$ if and only if $A \subseteq Cl(Int(Cl(A)))$.

The family of all semi$-$preopen sets in a topological space (X, τ) is denoted by $SPO(X, \tau)$.

Definition 2.1.12. [5] Let (X, τ) be a topological space and $A \subseteq X$. Then A is called $locally-closed$ if $A = U \cap B$ when U is open and B is closed in X.

The family of all locally closed sets in a topological space (X, τ) is denoted by $\mathcal{LC}\,(X, \tau)$.

Definition 2.1.13. [20] Let (X, τ) and $(Y, \mathcal{U})$ be topological spaces and $f : (X, \tau) \to (Y, \mathcal{U})$. Then f is called $continuous$ if $f^{-1}(V) \in \tau$ for each $V \in \mathcal{U}$.

Definition 2.1.14. [17] Let (X, τ) and $(Y, \mathcal{U})$ be topological spaces and $f : (X, \tau) \to (Y, \mathcal{U})$. Then f is called $semi-continuous$ if $f^{-1}(V) \in SO(X, \tau)$ for each $V \in \mathcal{U}$.

Definition 2.1.15. [9] Let (X, τ) and $(Y, \mathcal{U})$ be topological spaces and $f : (X, \tau) \to (Y, \mathcal{U})$. Then f is called $\alpha-continuous$ if $f^{-1}(V) \in \tau^{\alpha}$ for each $V \in \mathcal{U}$.

Definition 2.1.16. [9] Let (X, τ) and $(Y, \mathcal{U})$ be topological spaces and $f : (X, \tau) \to (Y, \mathcal{U})$. Then f is called $spr-continuous$ if $f^{-1}(V) \in SPO(X, \tau)$ for each $V \in \mathcal{U}$.

Definition 2.1.17. [9] Let (X,τ) and $(Y,\mathcal{U})$ be topological spaces and $f:(X,\tau)\to(Y,\mathcal{U})$. Then f is called $\mathcal{LC}-continuous$ if $f^{-1}(V)\in\mathcal{LC}(X,\tau)$ for each $V\in\mathcal{U}$.

Definition 2.1.18. [7] Let (X,τ) be a topological space and $A\subseteq X$. Then A is called $regular\ closed$ if and only if $A=Cl(Int(A))$.

The family of all regular closed sets in a topological space (X,τ) is denoted by $RC\,(X,\tau)$.

Definition 2.1.19. [9] Let (X,τ) be a topological space and $M\subseteq X$. Then M is called an $\mathcal{A}-set$ if $M=U\cap B$ when U is open and B is regular closed in X.

The family of all $\mathcal{A}-$ sets in a topological space (X,τ) is denoted by $\mathcal{A}\,(X,\tau)$.

Example 2.1.20. Let $X=\{1,2,3\}$ and $\tau=\{\emptyset,\{1\},\{2\},\{1,2\},\{2,3\},X\}$.
By the definition of τ, we get the following table.

M	$Int(M)$	$Cl(Int(M))$
$\emptyset$	$\emptyset$	$\emptyset$
$\{1\}$	$\{1\}$	$\{1\}$
$\{2\}$	$\{2\}$	$\{2,3\}$
$\{3\}$	$\emptyset$	$\emptyset$
$\{1,2\}$	$\{1,2\}$	X
$\{1,3\}$	$\{1\}$	$\{1\}$
$\{2,3\}$	$\{2,3\}$	$\{2,3\}$
X	X	X

Hence $RC(X,\tau)=\{\emptyset,\{1\},\{2,3\},X\}$. Thus $\mathcal{A}\,(X,\tau)=\{\emptyset,\{1\},\{2\},\{1,2\},\{2,3\},X\}$.

Definition 2.1.21. [22] Let (X,τ) and $(Y,\mathcal{U})$ be topological spaces and $f:(X,\tau)\to(Y,\mathcal{U})$. Then f is called $\mathcal{A}-continuous$ if $f^{-1}(V)\in\mathcal{A}(X,\tau)$ for each $V\in\mathcal{U}$.

Definition 2.1.22. [22] Let (X,τ) be a topological space and $A\subseteq X$. Then A is called a $t-set$ if and only if $Int(A)=Int(Cl(A))$.

The family of all $t-$ sets in a topological space (X,τ) is denoted by $t(X,\tau)$.

Proposition 2.1.23. [22] Let (X,τ) be a topological space and $A\subseteq X$. Then A is a $t-$ set if and only if A is semi$-$closed.

Definition 2.1.24. [8] Let (X, τ) be a topological space and $A \subseteq X$. Then A is called a $\mathcal{B} - set$ if $A = U \cap B$ when U is open and B is a $t-$ set.

The family of all $\mathcal{B}-$ sets in a topological space (X, τ) is denoted by $\mathcal{B}(X, \tau)$.

Proposition 2.1.25. [8] Let (X, τ) be a topological space.
Then $\mathcal{A}(X, \tau) \subseteq \mathcal{LC}(X, \tau) \subseteq \mathcal{B}(X, \tau)$.

Proposition 2.1.26. [8] Let (X, τ) be a topological space and A is open in (X, τ).
Then $Cl(A)$ is regular closed.

Definition 2.1.27. [8] Let (X, τ) be a topological space and $A \subseteq X$. Then A is called a $\mathcal{C} - set$ if $A = U \cap B$ when U is open and B is pre$-$closed.

The family of all $\mathcal{C}-$ sets in a topological space (X, τ) is denoted by $\mathcal{C}(X, \tau)$.

Proposition 2.1.28. [8] Let (X, τ) be a topological space and $A \subseteq X$. Then $\tau \subseteq \mathcal{C}(X, \tau)$.

Proposition 2.1.29. [8] Let (X, τ) be a topological space and $A \subseteq X$. If A is pre$-$closed, then A is a $\mathcal{C}-$ set.

Proposition 2.1.30. [8] Let (X, τ) be a topological space and $A \subseteq X$. If A is closed, then A is a $\mathcal{C}-$ set.

Proposition 2.1.31. [8] Let (X, τ) be a topological space. Then $\mathcal{A}(X, \tau) \subseteq \mathcal{LC}(X, \tau) \subseteq \mathcal{C}(X, \tau)$.

Proposition 2.1.32. [8] Let (X, τ) be a topological space and $A \subseteq X$. Then $pcl(A)$ is pre$-$closed.

Lemma 2.1.33. [8] Let (X, τ) be a topological space and $H \subseteq X$. Then the following statements are equivalent:

(1) $H \in \mathcal{C}(X, \tau)$;

(2) There exists an open set U in (X, τ) such that $H = U \cap pcl(H)$.

Lemma 2.1.34. [8] Let (X, τ) be a topological space and $H \subseteq X$. Then the following statements are equivalent:

(1) There exists an open set U in (X, τ) such that $H = U \cap Cl(Int(H)$;

(2) $H \in \mathcal{C}(X, \tau) \cap SO(X, \tau)$.

Theorem 2.1.35. [8] Let (X,τ) be a topological space.
Then $\mathcal{A}(X,\tau) = SO(X,\tau) \cap \mathcal{LC}(X,\tau)$.

Theorem 2.1.36. [8] Let (X,τ) be a topological space.
Then $\mathcal{A}(X,\tau) = \mathcal{C}(X,\tau) \cap\ SO(X,\tau)$.

Definition 2.1.37. [8] Let (X,τ) and $(Y,\mathcal{U})$ be topological spaces and $f : (X,\tau) \to (Y,\mathcal{U})$. Then f is called $\mathcal{C}-continuous$ if $f^{-1}(V) \in \mathcal{C}(X,\tau)$ for each $V \in \mathcal{U}$.

Theorem 2.1.38. [8] Let (X,τ) and $(Y,\mathcal{U})$ be topological spaces and $f : (X,\tau) \to (Y,\mathcal{U})$. Then f is $\mathcal{A}-$continuous if and only if it is $semi-$continuous and $\mathcal{C}-$continuous.

Proposition 2.1.39. [22] Let (X,τ) and $(Y,\mathcal{U})$ be topological spaces and $f : (X,\tau) \to (Y,\mathcal{U})$. Then f is continuous if and only if it is $\alpha-$continuous and $\mathcal{A}-$continuous.

Corollary 2.1.40. [8] Let (X,τ) and $(Y,\mathcal{U})$ be topological spaces and $f : (X,\tau) \to (Y,\mathcal{U})$. Then f is continuous if and only if it is $\alpha-$continuous and $\mathcal{C}-$continuous.

Proposition 2.1.41. [5] Let (X,τ) be a topological space and $H \subseteq X$. Then the following statements are equivalent:

(1) $H \in \mathcal{LC}(X,\tau)$;

(2) There exists an open set U in (X,τ) such that $H = U \cap Cl(H)$.

Proposition 2.1.42. [8] Let (X,τ) be a topological space. Then $SO(X,\tau) \subseteq SPO(X,\tau)$.

Theorem 2.1.43. [8] Let (X,τ) be a topological space. Then $\mathcal{A}(X,\tau) = SPO(X,\tau) \cap \mathcal{LC}(X,\tau)$.

Theorem 2.1.44. [8] Let (X,τ) and $(Y,\mathcal{U})$ be topological spaces and $f : (X,\tau) \to (Y,\mathcal{U})$. Then f is continuous if and only if it is $spr-$continuous and $\mathcal{LC}-$continuous.

2.2 Minimal structure spaces

In this section, we introduce the $m-$structure and the $m-$operator notions. Also, we define some important subsets associated to these concepts.

Definition 2.2.1. [17] Let X be a nonempty set and $P(X)$ be the power set of X. A subfamily m_X of $P(X)$ is called a $minimal\ structure$ (briefly $m-structure$) on X if $\emptyset \in m_X$ and $X \in m_X$.

The pair (X, m_X), we denote a nonempty set X with an $m-$structure m_X on X and it is called a $minimalstructurespace$ (briefly $m-space$). Each member of m_X is said to be m_X-open and the complement of an m_X- open set is said to be $m_X-closed$.

Definition 2.2.2. [17] Let X be a nonempty set and m_X an $m-$structure on X. For a subset A of X, the $m_X - interior$ of A and the $m_X - closure$ of A with respect to m_X are defined as follows:

(1) $m_X Int(A) = \bigcup\{U : U \subseteq A, U \in m_X\}$;

(2) $m_X Cl(A) = \bigcap\{F : A \subseteq F, X \setminus F \in m_X\}$.

Lemma 2.2.3. [14] Let X be a nonempty set and m_X an $m-$structure on X. For any subsets A and B of X, the following properties hold:

(1) $m_X Cl(X \setminus A) = X \setminus m_X Int(A)$ and $m_X Int(X \setminus A) = X \setminus m_X Cl(A)$;

(2) If $(X \setminus A) \in m_X$, then $m_X Cl(A) = A$ and if $A \in m_X$, then $m_X Int(A) = A$;

(3) $m_X Cl(\emptyset) = \emptyset, m_X Cl(X) = X, m_X Int(\emptyset) = \emptyset$ and $m_X Int(X) = X$;

(4) If $A \subseteq B$, then $m_X Cl(A) \subseteq m_X Cl(B)$ and $m_X Int(A) \subseteq m_X Int(B)$;

(5) $A \subseteq m_X Cl(A)$ and $m_X Int(A) \subseteq A$;

(6) $m_X Cl(m_X Cl(A)) = m_X Cl(A)$ and $m_X Int(m_X Int(A)) = m_X Int(A)$;

(7) $m_X Int(A \cap B) = m_X Int(A) \cap m_X Int(B)$ and
$m_X Int(A) \cup m_X Int(B) \subseteq m_X Int(A \cup B)$;

(8) $m_X Cl(A \cup B) = m_X Cl(A) \cup m_X Cl(B)$ and $m_X Cl(A \cap B)$
$\subseteq m_X Cl(A) \cap m_X Cl(B)$.

Definition 2.2.4. [15] An $m-$structure m_X on a nonempty set X is said to have property $\mathfrak{B}$ if the union of any family of subsets belonging to m_X belongs to m_X.

Lemma 2.2.5. [17]Let X be a nonempty set and m_X is an $m-$structure on X satisfying property $\mathfrak{B}$. For $A \subseteq X$ the following properties hold:

(1) $A \in m_X$ if and only if $m_X Int(A) = A$,

(2) A is m_X-closed if and only if $m_X Cl(A) = A$,

(3) $m_X Int(A)$ is m_X-open and $m_X Cl(A)$ is m_X-closed.

Lemma 2.2.6. [15] Let X be a nonempty set and m_X is an $m-$structure on X. For any subset A of X, $x \in m_XCl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.

Definition 2.2.7. [2] Let (X, m_X) be an $m-$space and $R \subseteq X$. Then R is called m_X-regular closed if and only if $R = m_XCl(m_XInt(R))$.

The family of all m_X-regular closed sets in an $m-$space (X, m_X) is denoted by $RC\,(X, m_X)$.

Definition 2.2.8. [19] A subset A of an $m-$space (X, m_X) is called an $m_X - preopen$ set if $A \subseteq m_XInt(m_XCl(A))$ and an $m_X - preclosed$ set if $m_XCl(m_XInt(A)) \subseteq A$.

The family of all m_X-preopen sets in an $m-$space (X, m_X) is denoted by $PO(X, m_X)$, and m_X-preclosed sets in an $m-$space (X, m_X) is denoted by $PC(X, m_X)$.

Definition 2.2.9. [19] A subset A of an $m-$space (X, m_X) is called an $m_X - semi - open$ if $A \subseteq m_XCl(m_XInt(A))$ and an $m_X - semi - closed$ if $m_XInt(m_XCl(A)) \subseteq A$.

The family of all m_X-semi$-$open in an $m-$space (X, m_X) is denoted by $SO(X, m_X)$, and m_X-semi$-$closed in an $m-$space (X, m_X) is denoted by $SC(X, m_X)$.

Definition 2.2.10. [19] Let (X, m_X) be an $m-$space and $A \subseteq X$, the $m_X - preclosure$ of A is denoted by $m_Xpcl(A)$ is defined as the intersection of all m_X-preclosed of (X, m_X) containing A.

Proposition 2.2.11. [19] Let (X, m_X) be an $m-$space and $A, B \subseteq X$. If $A \subseteq B$, then $m_Xpcl(A) \subseteq m_Xpcl(B)$.

Proposition 2.2.12. [19] Let (X, m_X) be an $m-$space and $A \subseteq X$. If m_X satisfies the property $\mathfrak{B}$. Then $m_Xpcl(A) = A \cup m_XCl(m_XInt(A))$.

2.3 Biminimal structure spaces

In this section, we introduce the $bim-$space and the $bim-$operator notions. Also, we define some important subsets associated to these concepts. This section discusses some properties of biminimal structure spaces.

Definition 2.3.1. [3] Let X be a nonempty set and m_X^1, m_X^2 be $m-$ structures on X. A triple (X, m_X^1, m_X^2) is called a *biminimal structure space* (briefly $bim - space$).

Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$. The $m_X-closure$ and $m_X - interior$ of A with respect to m_X^i are denoted by $m_X^iCl(A)$ and $m_X^iInt(A)$ respectively, for $i = 1, 2$.

Each member of m_X^i is said to be an $m_X^i - open$ set and the complement of an m_X^i-open set is said to be $m_X^i - closed$, for $i = 1, 2$.

Definition 2.3.2. [3] Let (X, m_X^1, m_X^2) be a biminimal structure space and Y be a subset of X. Define minimal structures m_Y^1, m_Y^2 on Y as follows:

$m_Y^1 = \{A \cap Y \mid A \in m_X^1\}$ and $m_Y^2 = \{B \cap Y \mid B \in m_X^2\}$. A triple (X, m_Y^1, m_Y^2) is called a $biminimal\ structure\ subspace$ (briefly $bim - subspace$) of (X, m_X^1, m_X^2).

Definition 2.3.3. [4] A subset A of biminimal structure spaces (X, m_X^1, m_X^2) is said to be

(1) $(i, j)m_X - regular\ open$ if $A = m_X^iInt(m_X^jCl(A))$, where $i, j = 1$ or 2 and $i \neq j$;

(2) $(i, j)m_X - semi - open$ if $A \subseteq m_X^iCl(m_X^jInt(A))$, where $i, j = 1$ or 2 and $i \neq j$;

(3) $(i, j)m_X - preopen$ if $A \subseteq m_X^iInt(m_X^jCl(A))$, where $i, j = 1$ or 2 and $i \neq j$;

(4) $(i, j)m_X - \alpha - open$ if $A \subseteq m_X^iInt(m_X^jCl(m_X^iInt(A)))$, where $i, j = 1$ or 2 and $i \neq j$;

The complement of an $(i, j)m_X-$ regular open (resp.($(i, j)m_X-$ semi-open, $(i, j)m_X-$ preopen, $(i, j)m_X - \alpha-$ open) set is called $(i, j)m_X - regular\ closed$ (resp. ($(i, j)m_X - semi - closed$, $(i, j)m_X - preclosed$, $(i, j)m_X - \alpha - closed$).

Lemma 2.3.4. [4] Let (X, m_X^1, m_X^2) be a biminimal structure space and A be a subset of X. Then

(1) A is $(i, j)m_X - regular\ closed$ if and only if $A = m_X^iCl(m_X^jInt(A))$;

(2) A is $(i, j)m_X - semi - closed$ if and only if $m_X^iInt(m_X^jCl(A)) \subseteq A$;

(3) A is $(i, j)m_X - preclosed$ if and only if $m_X^iCl(m_X^jInt(A)) \subseteq A$;

(4) A is $(i, j)m_X - \alpha - closed$ if and only if $m_X^iCl(m_X^jInt(m_X^iCl(A))) \subseteq A$.

Definition 2.3.5. [4] Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure space. A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be $(i, j) - M - continuous$ at a point $x \in X$ and each $V \in m_Y^i$ containing $f(x)$, there exists $U \in m_X^j$ containing x such that $f(U) \subseteq V$, where $i, j = 1$ or 2 and $i \neq j$.

A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be $(i, j) - M - continuous$ if it has this property at each point $x \in X$.

Theorem 2.3.6. [4] For a function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$, the following properties are equivalen:

(1) f is $(i, j) - M-$continuous;

(2) $f^{-1}(V) = m_X^j Int(f^{-1}(V))$ for every $V \in m_Y^i$;

(3) $f(m_X^i Cl(A)) \subseteq m_Y^i Cl(f(A))$ for every subset A of X;

(4) $m_X^j Cl(f^{-1}(B)) \subseteq f^{-1}(m_Y^i Cl(B))$ for every subset B of Y;

(5) $f^{-1}(m_Y^i Int(B)) \subseteq m_X^j Int(f^{-1}(B))$ for every subset B of Y;

(6) $m_X^j Cl(f^{-1}(F)) = f^{-1}(F)$ for every m_Y^i-closed set F of Y.

CHAPTER 3

$\mathcal{A}-$ SETS IN BIMINIMAL STRUCTURE SPACES

In this section, we introduce the concept of $\mathcal{A}-$sets in biminimal structure spaces and study some fundamental properties of $\mathcal{A}-$sets in biminimal structure spaces.

3.1 $\mathcal{A}-$sets in minimal structure spaces

In this section, we will introduce the notion of $\mathcal{A}-$sets in minimal structure spaces and investigate some of their properties.

Definition 3.1.1. Let (X, m_X) be an $m-$space. A subset M of X is said to be an $m_X - \mathcal{A} - set$ if there exist G and R such that $M = G \cap R$ when G is m_X-open and R is m_X-regular closed.

The family of all $m_X-\mathcal{A}-$sets in an $m-$space (X, m_X) is denoted by $\mathcal{A}(X, m_X)$.

Example 3.1.2. Let $X = \{1, 2, 3\}$. Define an $m-$structure m_X on X as follows:
$m_X = \{\emptyset, \{2\}, \{1, 2\}, \{1, 3\}, X\}$. Then $RC(X, m_X) = \{\emptyset, \{2\}, \{1, 3\}, X\}$ and
$\mathcal{A}(X, m_X) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, X\}$.

Definition 3.1.3. Let (X, m_X) be an $m-$space and $A \subseteq X$, then A is said to be an $m_X - t - set$ if $m_XInt(A) = m_XInt(m_XCl(A))$.

The family of all $m_X - t-$sets in an $m-$space (X, m_X) is denoted by $t(X, m_X)$.

Example 3.1.4. Let $X = \{1, 2, 3\}$ and define $m_X = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$ be an $m-$structure on X. It follows that $t(X, m_X) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}, X\}$.

Proposition 3.1.5. Let (X, m_X) be an $m-$space and $R \subseteq X$. If R is m_X- regular closed then R is an $m_X - t-$set.

Proof. Let R be an m_X- regular closed. Then $R = m_XCl(m_XInt(R))$.
Consequently, $m_XCl(R) = m_XCl(m_XCl(m_XInt(R)))$.
Thus $m_XInt(m_XCl(R)) = m_XInt(m_XCl(m_XInt(R)))$. Hence $m_XInt(m_XCl(R)) = m_XInt(R)$. Therefore, R is an $m_X - t-$set. □

The converse is not true as can be seen from the following example.

Example 3.1.6. Let $X = \{1, 2, 3\}$. Define $m-$structures m_X on X as follows : $m_X = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$. Thus $t(X, m_X) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}, X\}$ and $RC(X, m_X) = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$.
We can see that $t(X, m_X)$ is not $RC(X, m_X)$.

3.2 $\mathcal{A}-$sets in biminimal structure spaces

In this section, we will introduce the notion of $\mathcal{A}-$sets in biminimal structure spaces and investigate some of their properties.

Definition 3.2.1. A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be $(i, j)m_X - locally\ closed$ if there exist G and F such that $A = G \cap F$ when G is an m_X^i-open set G and F is an m_X^j-closed set, where $i, j = 1, 2$ and $i \neq j$.

The family of all $(i, j)m_X-$ locally closed sets in biminimal structure spaces (X, m_X^1, m_X^2) is denoted by $(i, j) - \mathcal{LC}\,(X, m_X^1, m_X^2)$, where $i, j = 1, 2$ and $i \neq j$.

Example 3.2.2. Let $X = \{a, b, c\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{b, c\}, X\}$ and $m_X^2 = \{\emptyset, \{c\}, X\}$. It follows that $\emptyset, \{a, b\}, X$ are m_X^2-closed. Thus $(1, 2) - \mathcal{LC}\,(X, m_X^1, m_X^2) = \{\emptyset, \{b\}, \{b, c\}, \{a, b\}, X\}$.

Lemma 3.2.3. Let S be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and let $i, j = 1, 2$ and $i \neq j$. If S is an $(i, j)m_X-$locally closed set then there exists an m_X^i-open set U such that $S = U \cap m_X^jCl(S)$.

Proof. Let S be a $(i, j)m_X-$locally closed set. Then there exist U and F such that $S = U \cap F$ where U is m_X^i-open and F is m_X^j- closed. Since $S = U \cap F, S \subseteq F$.
Thus $m_X^jCl(S) \subseteq m_X^jCl(F)$. Since F is m_X^j-closed, $m_X^jCl(S) \subseteq F$.
Then $U \cap m_X^jCl(S) \subseteq U \cap F = S$. Since $S \subseteq U$ and $S \subseteq m_X^jCl(S)$.
Then $S \subseteq U \cap m_X^jCl(S)$. Therefore, there exists an m_X^j- open set U such that $S = U \cap m_X^jCl(S)$. □

The converse is not true as can be seen the following example.

Example 3.2.4. Let $X = \{1, 2, 3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1\}, \{2\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{2, 3\}, X\}$. Set $S = \{3\}$. Since there exists $X \in m_X^1$ such that $S = X \cap m_X^2Cl(S)$ and $(1, 2) - \mathcal{LC}\,(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$. We see that S is not a $(1, 2) - \mathcal{LC}\,(X, m_X^1, m_X^2)$.

The converse of above lemma is true if m_X^j has property $\mathfrak{B}$ as following proposition.

Proposition 3.2.5. Let S be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and let m_X^j has property $\mathfrak{B}$, where $i, j = 1, 2$ and $i \neq j$. Then S is an $(i, j)m_X-$locally closed set iff there exists an m_X^i-open set U such that $S = U \cap m_X^j Cl(S)$.

Proof. $(\Rightarrow)$ By Lemma 3.2.3.

$(\Leftarrow)$ Let $S = U \cap m_X^j Cl(S)$, for some $U \in m_X^i$. Since m_X^j has property $\mathfrak{B}$, $m_X^j Cl(S)$ is closed in (X, m_X^j). Thus S is an $(i, j)m_X-$locally closed. □

Definition 3.2.6. Let (X, m_X^1, m_X^2) be a biminimal structure space. A subset M of X is said to be an $(i, j)m_X - \mathcal{A} - set$ if there exist G and R, such that $M = G \cap R$ when $G \in m_X^i$ and R is m_X^j-regular closed, where $i, j = 1, 2$ and $i \neq j$.

The family of all $(i, j)m_X - \mathcal{A}-$sets in a biminimal structure space (X, m_X^1, m_X^2) is denoted by $(i, j) - \mathcal{A}\,(X, m_X^1, m_X^2)$, where $i, j = 1, 2$ and $i \neq j$.

Example 3.2.7. Let $X = \{1, 2, 3\}$. Define $m_X^1 = \{\emptyset, \{1, 2\}, \{1, 3\}, X\}$ and $m_X^2 = \{\emptyset, \{2\}, \{1, 2\}, X\}$ which are $m-$structures on X. It follows that $RC(X, m_X^2) = \{\emptyset, X\}$. Thus $(1, 2) - \mathcal{A}\,(X, m_X^1, m_X^2) = \{\emptyset, \{1, 2\}, \{1, 3\}, X\}$.

Remark. The intersection of two $(i, j)m_X - \mathcal{A}-$sets may not be an $(i, j)m_X - \mathcal{A}-$set as shown in the next example.

Example 3.2.8. Let $X = \{1, 2, 3\}$. Define $m_X^1 = \{\emptyset, \{1, 2\}, \{1, 3\}, X\}$ and $m_X^2 = \{\emptyset, \{2\}, \{1, 2\}, X\}$ which are $m-$structures on X.
It follows that $\{1, 2\}$ and $\{1, 3\}$ are $(1, 2)m_X - \mathcal{A}-$sets. But $\{1, 2\} \cap \{1, 3\}$ is not a $(1, 2)m_X - \mathcal{A}-$set.

Remark. The union of two $(i, j)m_X - \mathcal{A}-$sets may not be an $(i, j)m_X - \mathcal{A}-$set as shown in the next example.

Example 3.2.9. Let $X = \{1, 2, 3\}$. Define $m_X^1 = \{\emptyset, \{1\}, \{2\}, \{2, 3\}, X\}$, $m_X^2 = \{\emptyset, \{1\}, \{3\}, \{2, 3\}, X\}$, which are $m-$structures on X. It follows that $RC(X, m_X^2) = \{\emptyset, \{1\}, \{2, 3\}, X\}$. Thus $(1, 2) - \mathcal{A}\,(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{2, 3\}, X\}$. Consequently $\{1\}$ and $\{2\}$ are $(1, 2)m_X - \mathcal{A}-$sets. But $\{1\} \cup \{2\}$ is not a $(1, 2)m_X - \mathcal{A}-$set.

Lemma 3.2.10. Let (X, m_X^1, m_X^2) be a biminimal structure space m_X^j has the property $\mathfrak{B}$. If a subset M of X is an $(i,j)m_X - \mathcal{A}-$set, then M is $(i,j)m_X-$locally closed, where $i,j = 1,2$ and $i \neq j$.

Proof. Let M is an $(i,j)m_X - \mathcal{A}-$set. Then there exist G and R such that $M = G \cap R$ where G is m_X^i- open and R is m_X^j-regular closed. Since R is m_X^j-regular closed, $R = m_X^j Cl(m_X^j Int(R))$. But m_X^j has the property $\mathfrak{B}$ then $m_X^j Cl(m_X^j Int(R))$ is closed. Hence R is m_X^j closed. It follows that M is an $(i,j)m_X-$locally closed. □

The converse of Lemma 3.2.10, is not true, as shown in the next example.

Example 3.2.11. Let $X = \{1,2,3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows : $m_X^1 = \{\emptyset, \{1\}, \{2\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{2,3\}, X\}$. Thus $(1,2) - \mathcal{A}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{2,3\}, X\}$ and $(1,2) - \mathcal{LC}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,3\}, \{2,3\}, X\}$.
We can see that $\{3\}$ is an $(1,2) - \mathcal{LC}(X, m_X^1, m_X^2)$ but it is not a $(1,2) - \mathcal{A}(X, m_X^1, m_X^2)$.

The converse of Lemma 3.2.10, is true if $m_X^j \subseteq m_X^i$ and m_X^j has the property $\mathfrak{B}$ as the following proposition.

Proposition 3.2.12. Let (X, m_X^1, m_X^2) be a biminimal structure space and $m_X^j \subseteq m_X^i$ has the property $\mathfrak{B}$. If a subset M of X is both $(i,j)m_X-$semi$-$open and $(i,j)m_X-$locally closed, then M is an $(i,j)m_X - \mathcal{A}-$set, where $i,j = 1,2$ and $i \neq j$.

Proof. Let M be both $(i,j)m_X-$semi$-$open and $(i,j)m_X-$locally closed.
It follows that $M \subseteq m_X^i Cl(m_X^j Int(M))$ and there exists an m_X^i-open set U such that $M = U \cap m_X^j Cl(M)$. Since $m_X^j Cl(M) \subseteq m_X^j Cl(m_X^i Cl(m_X^j Int(M))) \subseteq m_X^j Cl(m_X^j Cl(m_X^j Int(M))) \subseteq m_X^j Cl(m_X^j Int(M))$. But $m_X^j Cl(m_X^j Int(M)) \subseteq m_X^j Cl(M)$, hence $m_X^j Cl(M) = m_X^j Cl(m_X^j Int(M))$.
We will show that $m_X^j Cl(m_X^j Int(M))$ is regular closed.
Since $m_X^j Int(M) = m_X^j Int(m_X^j Int(M)) \subseteq m_X^j Int(m_X^j Cl(m_X^j Int(M)))$.
It follows that $m_X^j Cl(m_X^j Int(M)) \subseteq m_X^j Cl(m_X^j Int(m_X^j Cl(m_X^j Int(M))))$.
Since $m_X^j Int(m_X^j Cl(m_X^j Int(M))) \subseteq m_X^j Cl(m_X^j Int(M))$.
Then $m_X^j Cl(m_X^j Int(m_X^j Cl(m_X^j Int(M)))) \subseteq m_X^j Cl(m_X^j Cl(m_X^j Int(M))) = m_X^j Cl(m_X^j Int(M))$. Thus $m_X^j Cl(m_X^j Int(M)) = m_X^j Cl(m_X^j Int(m_X^j Cl(m_X^j Int(M))))$.

Hence $m_X^jCl(m_X^jInt(M))$ is m_X^j regular closed. Consequently $m_X^jCl(M)$ is m_X^j regular closed. Therefore, M is an $(i,j)m_X-\mathcal{A}-$set. $\square$

Definition 3.2.13. Let (X,m_X^1,m_X^2) be a biminimal structure space and $A\subseteq X$. Then A is said to be an $(i,j)m_X-t-set$ if $m_X^iInt(A)=m_X^iInt(m_X^jCl(A))$, where $i,j=1,2$ and $i\neq j$.

The family of all $(i,j)m_X-t-$sets in a biminimal structure spaces (X,m_X^1,m_X^2) is denoted by $(i,j)-t(X,m_X^1,m_X^2)$ for $i,j=1,2$ and $i\neq j$.

Example 3.2.14. Let $X=\{1,2,3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows: $m_X^1=\{\emptyset,\{1\},\{3\},\{2,3\},X\}$ and $m_X^2=\{\emptyset,\{1\},\{1,2\},X\}$.
Thus $(1,2)-t(X,m_X^1,m_X^2)=\{\emptyset,\{3\},\{2,3\},X\}$.

Theorem 3.2.15. Let (X,m_X^1,m_X^2) be a biminimal structure space and $A\subseteq X$. Then A is an $(i,j)m_X-t-$set if and only if A is $(i,j)m_X-$semi$-$closed, where $i,j=1,2$ and $i\neq j$.

Proof. $(\Rightarrow)$ Let A be an $(i,j)m_X-t-$ set. Then $m_X^iInt(A)=m_X^iInt(m_X^jCl(A))$. Thus $m_X^iInt(m_X^jCl(A))\subseteq A$. Hence A is $(i,j)m_X-$ semi$-$closed .

$(\Leftarrow)$ Let A be $(i,j)m_X-$semi$-$closed. Then $m_X^iInt(m_X^jCl(A))\subseteq A$. Thus $m_X^iInt(m_X^iInt(m_X^jCl(A)))\subseteq m_X^iInt(A)$. Hence $m_X^iInt(m_X^jCl(A))\subseteq m_X^iInt(A)$. Since $m_X^iInt(A)\subseteq m_X^iInt(m_X^jCl(A))$. Thus $m_X^iInt(A)=m_X^iInt(m_X^jCl(A))$. Hence A is an $(i,j)m_X-t-$set. $\square$

Definition 3.2.16. Let (X,m_X^1,m_X^2) be a biminimal structure space and $A\subseteq X$. Then A is said to be an $(i,j)m_X-\mathcal{B}-set$ if $A=U\cap T$, when U is an m_X^i-open set and T is an m_X^j-t-set, where $i,j=1,2$ and $i\neq j$.

The family of all $(i,j)m_X-\mathcal{B}-$sets in a biminimal structure space (X,m_X^1,m_X^2) is denoted by $(i,j)-\mathcal{B}(X,m_X^1,m_X^2)$, where $i,j=1,2$ and $i\neq j$.

Example 3.2.17. Let $X=\{1,2,3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows: $m_X^1=\{\emptyset,\{1\},\{2\},\{2,3\},X\}$ and $m_X^2=\{\emptyset,\{1\},\{3\},\{2,3\},X\}$.
Then $\{\emptyset,\{1\},\{2\},\{1,2\},\{2,3\},X\}$ are m_X^2-t-sets.
Therefore, $(1,2)-\mathcal{B}(X,m_X^1,m_X^2)=\{\emptyset,\{1\},\{2\},\{1,2\},\{2,3\},X\}$.

Theorem 3.2.18. Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$. If A is an $(i,j)m_X - \mathcal{A}-$set, then A is an $(i,j)m_X - \mathcal{B}-$set for all i, j= 1, 2 and $i \neq j$.

Proof. Let A be an $(i,j)m_X - \mathcal{A}-$set. Then there exist G and R such that $A = G \cap R$ where G is m_X^i-open in (X, m_X^i) and R is an m_X^j- regular closed. By Proposition 3.1.5, R is an $m_X^j - t-$set. Hence A is an $(i,j)m_X - \mathcal{B}-$set. □

The converse is not true as can be seen from the following example.

Example 3.2.19. Let $X = \{1,2,3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows : $m_X^1 = \{\emptyset, \{1\}, \{3\}, \{2,3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, X\}$. Thus $(1,2) - \mathcal{A}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{3\}, \{1,3\}, \{2,3\}, X\}$ and $(1,2) - \mathcal{B}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,3\}, \{2,3\}, X\}$. We can see that $\{2\}$ is a $(1,2)m_X - \mathcal{B}-$set but it is not a $(1,2)m_X - \mathcal{A}-$set.

Definition 3.2.20. Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$. Then A is said to be an $(i,j)m_X - \mathcal{C} - set$ if $A = U \cap B$, when U is an m_X^i-open and B is m_X^j- preclosed, where $i, j = 1, 2$ and $i \neq j$.

The family of all $(i,j) - \mathcal{C}-$sets in a biminimal structure space (X, m_X^1, m_X^2) is denoted by $(i,j) - \mathcal{C}\,(X, m_X^1, m_X^2)$, where $i, j = 1, 2$ and $i \neq j$.

Example 3.2.21. Let $X = \{1,2,3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows : $m_X^1 = \{\emptyset, \{1\}, \{2\}, \{2,3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{3\}, \{2,3\}, X\}$.
Thus $\emptyset, \{1\}, \{2\}, \{1,2\}, \{2,3\}, X$ are preclosed in (X, m_X^2).
Therefore, $(1,2) - \mathcal{C}\,(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{2,3\}, X\}$.

Theorem 3.2.22. Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$. If A is an $(i,j)m_X - \mathcal{A}-$set, then it is an $(i,j)m_X - \mathcal{C}-$set for all i, j = 1, 2 and $i \neq j$.

Proof. Let A be an $(i,j)m_X - \mathcal{A}-$set. Then there exist G and R such that $A = G \cap R$ where G is m_X^i-open and R is m_X^j- regular closed. Since $R = m_X^j Cl(m_X^j Int(R))$, thus $m_X^j Cl(m_X^j Int(R)) \subseteq R$. Hence R is an m_X^j- preclosed.
Therefore, A is an $(i,j)m_X - \mathcal{C}-$set. □

The converse is not true as can be seen from the following example.

Example 3.2.23. Let $X = \{1,2,3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows :$m_X^1 = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\} = m_X^2$. Thus $(1,2)-\mathcal{A}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$ and $(1,2)-\mathcal{C}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,3\}, \{2,3\}, X\}$. We can see that $\{3\}$ is a $(1,2)m_X - \mathcal{C}-$set but it is not a $(1,2)m_X - \mathcal{A}-$set.

Proposition 3.2.24. Let (X, m_X^1, m_X^2) be a biminimal structure space and M be a subset of X. If M is an $(i,j)m_X-$locally closed set, then it is also an $(i,j)m_X - \mathcal{B}-$set, where $i,j = 1,2$ and $i \neq j$.

Proof. Let M be $(i,j)m_X-$locally closed set. Then there exist U and B such that $M = U \cap B$ where U is m_X^i-open and B is m_X^j-closed. Since B is m_X^j-closed, $B = m_X^jCl(B)$. Thus $m_X^jInt(B) = m_X^jInt(m_X^jCl(B))$. Hence B is an $m_X^j - t-$set. Thus M is an $(i,j)m_X - \mathcal{B}-$set. □

In general, an $(i,j)m_X - \mathcal{B}-$set need not be $(i,j)m_X-$locally closed set, as can be seen from the following example.

Example 3.2.25. Let $X = \{1,2,3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows : $m_X^1 = \{\emptyset, \{1\}, \{3\}, \{2,3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, X\}$. Thus $(1,2)-\mathcal{B}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,3\}, \{2,3\}, X\}$ and $(1,2)-\mathcal{LC}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{3\}, \{1,3\}, \{2,3\}, X\}$. We can see that $\{2\}$ is a $(1,2)m_X - \mathcal{B}-$set but it is not $(1,2)m_X-$locally closed set.

Proposition 3.2.26. Let (X, m_X^1, m_X^2) be a biminimal structure space and M be a subset of X. If M is an $(i,j)m_X-$locally closed set, then it is also an $(i,j)m_X - \mathcal{C}-$set, where $i,j = 1,2$ and $i \neq j$.

Proof. Let M be an $(i,j)m_X-$locally closed set. Then there exist U and B such that $M = U \cap B$ where U is an m_X^i-open in (X, m_X^i) and B is an m_X^j-closed. It follows that $m_X^jCl(m_X^jInt(B)) \subseteq m_X^jCl(B) = B$. Then B is an m_X^j-preclosed. Hence M is an $(i,j)m_X - \mathcal{C}-$set. □

In general, an $(i,j)m_X - \mathcal{C}-$set is not $(i,j)m_X-$locally closed set, as can be seen from the following example.

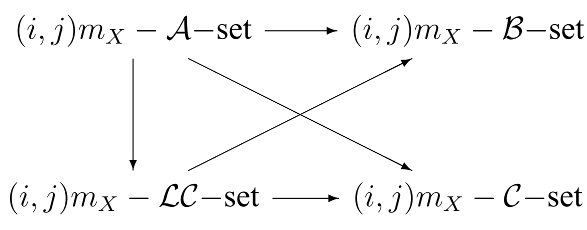
Example 3.2.27. Let $X = \{1, 2, 3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows : $m_X^1 = \{\emptyset, \{1\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{2, 3\}, X\}$.
Thus $(1, 2) - \mathcal{C}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, X\}$ and $(1, 2) - \mathcal{LC}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{1, 3\}, \{2, 3\}, X\}$.
We can see that $\{2\}$ is a $(1, 2)m_X - \mathcal{C}-$set but it is not a $(1, 2)m_X-$locally closed set.

Moreover, an $(i, j)m_X - \mathcal{B}-$set and an $(i, j)m_X - \mathcal{C}-$set are independent as can be seen from the following examples.

Example 3.2.28. Let $X = \{1, 2, 3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows : $m_X^1 = \{\emptyset, \{1\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{2, 3\}, X\}$.
Thus $(1, 2) - \mathcal{B}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}, X\}$ and $(1, 2) - \mathcal{C}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, X\}$. We can see that $\{1, 2\}$ is a $(1, 2)m_X - \mathcal{C}-$set but it is not a $(1, 2)m_X - \mathcal{B}-$set.

Example 3.2.29. Let $X = \{1, 2, 3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{2\}, \{3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, X\}$. Thus $(1, 2) - \mathcal{B}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}, X\}$ and $(1, 2) - \mathcal{C}(X, m_X^1, m_X^2) = \{\emptyset, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}, X\}$. We can see that $\{1\}$ is a $(1, 2)m_X - \mathcal{B}-$set but it is not $(1, 2)m_X - \mathcal{C}-$set.

We can conclude the relation among an $(i, j)m_X - \mathcal{A}-$set, an $(i, j)m_X - \mathcal{B}-$set, an $(i, j)m_X - \mathcal{C}-$set, an $(i, j)m_X - \mathcal{LC}-$set as the following diagram.



Proposition 3.2.30. Let A be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and m_X^j has the property $\mathfrak{B}$. Then A is an $(i, j) - \mathcal{C}-$set iff $A = U \cap m_X^j pcl(A)$ for some $U \in m_X^i$, where $i, j = 1, 2$ and $i \neq j$.

Proof. ($\Rightarrow$) Let A be $(i, j)m_X - \mathcal{C}-$set. Then there exist U and B such that $A = U \cap B$ where U is m_X^i-open and B is m_X^j-preclosed. From $A \subseteq B$, $m_X^j pcl(A) \subseteq m_X^j pcl(B)$

by Proposition 2.2.12, $m_X^j pcl(B) = B \cup m_X^j Cl(m_X^j Int(B))$. As B is m_X^j-preclosed, $m_X^j Cl(m_X^j Int(B)) \subseteq B$. Hence $m_X^j pcl(B) = B$. Thus $m_X^j pcl(A) \subseteq B$. It follows that $U \cap m_X^j pcl(A) \subseteq U \cap B = A$. Since $A \subseteq U$ and $A \subseteq m_X^j pcl(A)$, $A \subseteq U \cap m_X^j pcl(A)$. Therefore $A = U \cap m_X^j pcl(A)$.

($\Leftarrow$) Let $A = U \cap m_X^j pcl(A)$ for some $U \in m_X^i$. Since $m_X^j pcl(A)$ is an m_X^j-preclosed. Therefore, A is an $(i,j)m_X - \mathcal{C}-$set. □

Proposition 3.2.31. Let A be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and m_X^j has the property $\mathfrak{B}$. Then $A = U \cap m_X^j Cl(m_X^j Int(A))$ for some $U \in m_X^i$ if and only if A is an m_X^j-semi$-$open and $(i,j)m_X - \mathcal{C}-$set, where $i,j = 1,2$ and $i \neq j$.

Proof. ($\Rightarrow$) Let $A = U \cap m_X^j Cl(m_X^j Int(A))$ for some $U \in m_X^i$. Then $A \subseteq m_X^j Cl(m_X^j Int(A))$. Thus A is m_X^j-semi$-$open. By Lemma 2.2.5, $m_X^j Cl(m_X^j Int(A))$ is m_X^j-closed. Since $m_X^j Int(m_X^j Cl(m_X^j Int(A))) \subseteq m_X^j Cl(m_X^j Int(A))$,
$m_X^j Cl(m_X^j Int(m_X^j Cl(m_X^j Int(A)))) \subseteq m_X^j Cl(m_X^j Int(A))$. Hence $m_X^j Cl(m_X^j Int(A)$ is m_X^j-preclosed. Then A is an $(i,j)m_X - \mathcal{C}-$set.

($\Leftarrow$) Let A be an m_X^j-semi$-$open and $(i,j)m_X - \mathcal{C}-$set. By proposition 3.2.30, $A = U \cap m_X^j pcl(A)$ for some $U \in m_X^i$. Since A is m_X^j-semi$-$open. Then $A \subseteq m_X^j Cl(m_X^j Int(A)$. Since m_X^j has the property $\mathfrak{B}$ and by Proposition 2.2.12, $m_X^j pcl(A) = A \cup m_X^j Cl(m_X^j Int(A))$. Thus $m_X^j pcl(A) = m_X^j Cl(m_X^j Int(A)$. Hence $A = U \cap m_X^j Cl(m_X^j Int(A))$ for some $U \in m_X^i$. □

Theorem 3.2.32. Let (X, m_X^1, m_X^2) be a biminimal structure space and m_X^j has the property $\mathfrak{B}$. If a subset M of X is an m_X^j-semi$-$open and $(i,j)m_X - \mathcal{C}-$set, then it is an $(i,j)m_X - \mathcal{A}-$set.

Proof. Let M be an m_X^j-semi$-$open and $(i,j)m_X - \mathcal{C}-$set. By Proposition 3.2.31, then $M = U \cap m_X^j Cl(m_X^j Int(M))$ for some $U \in m_X^i$. Since $m_X^j Cl(m_X^j Int(M))$ is m_X^j-regular closed. Therefore, M is an $(i,j)m_X - \mathcal{A}-$set. □

Definition 3.2.33. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be

(1) $(i,j) - semi - continuous$ if $f^{-1}(V) \in (i,j) - SO(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

(2) $(i,j)-\mathcal{LC}-continuous$ if $f^{-1}(V) \in (i,j)-\mathcal{LC}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

(3) $(i,j)-\mathcal{A}-continuous$ if $f^{-1}(V) \in (i,j)-\mathcal{A}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

Example 3.2.34. Let $X = \{1,2,3\}$ and $Y = \{a,b\}$.

Consider $m-$structures on X and Y as follows :

$m_X^1 = \{\emptyset, \{1,2\}, \{1,3\}, X\}$ and $m_X^2 = \{\emptyset, \{2\}, \{1,2\}, X\}$.

$m_Y^1 = \{\emptyset, \{a\}, \{a,b\}, Y\}$ and $m_Y^2 = \{\emptyset, \{a\}, \{b\}, Y\}$.

Let $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ and $f(\emptyset) = \emptyset$, $f(\{1\}) = \{a\}$, $f(\{2\}) = \{a\}$, $f(\{3\}) = \{b\}$, $f(X) = Y$.

Consider $\emptyset, \{a\}, \{a,b\}, Y \in m_Y^1$, we get $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(\{a\}) = \{1,2\}$, $f^{-1}(\{a,b\})$ $= \{1,3\}$, $f^{-1}(Y) = X$ are $(1,2)m_X-\mathcal{A}-$sets. Thus f is $(1,2)-\mathcal{A}-$continuous.

Proposition 3.2.35. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces and let $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ be a mapping. If f is $(i,j)-\mathcal{A}-$continuous then f is $(i,j)-\mathcal{LC}-$continuous.

Proof. Let f be $(i,j)-\mathcal{A}-$continuous and $V \in m_Y^i$.
Then $f^{-1}(V) \in (i,j)-\mathcal{A}(X, m_X^1, m_X^2)$. By Lemma 3.2.10, we have $f^{-1}(V) \in (i,j)-\mathcal{LC}(X, m_X^1, m_X^2)$. Hence f is $(i,j)-\mathcal{LC}-$continuous. □

Theorem 3.2.36. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces and m_X^j has the property $\mathfrak{B}$. If a mapping $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is $(i,j)-semi-$continuous and $(i,j)-\mathcal{LC}-$continuous then f is $(i,j)-\mathcal{A}-$continuous.

Proof. Let f be an $(i,j)-semi-$continuous and $(i,j)-\mathcal{LC}-$continuous and $V \in m_Y^i$. Then $f^{-1}(V) \in (i,j)-SO(X, m_X^1, m_X^2)$ and $f^{-1}(V) \in (i,j)-\mathcal{LC}(X, m_X^1, m_X^2)$. By Theorem 3.2.12, thus $f^{-1}(V) \in (i,j)-\mathcal{A}(X, m_X^1, m_X^2)$. Therefore, f is $(i,j)-\mathcal{A}-$continuous. □

Definition 3.2.37. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be $(i,j)-\mathcal{C}-continuous$ if $f^{-1}(V) \in (i,j)-\mathcal{C}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

Example 3.2.38. Let $X = \{1,2,3\}$ and $Y = \{a,b,c\}$.

Consider $m-$structures on X and Y as follows :

$m_X^1 = \{\emptyset, \{1\}, \{2\}, \{2,3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{3\}, \{2,3\}, X\}$.

$m_Y^1 = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, Y\}$ and $m_Y^2 = \{\emptyset, \{a\}, \{b\}, Y\}$.

Let $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ and $f(\emptyset) = \emptyset, f(\{1\}) = \{a\}, f(\{2\}) = \{b\}$, $f(\{3\}) = \{c\}, f(X) = Y$.

Consider $\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, Y \in m_Y^1$, we get $f^{-1}(\emptyset) = \emptyset, f^{-1}(\{a\}) = \{1\}$, $f^{-1}(\{b\}) = \{2\}, f^{-1}(\{a,b\}) = \{1,2\}, f^{-1}(\{b,c\}) = \{2,3\}, f^{-1}(Y) = X$ are $(1,2)m_X-$ $\mathcal{C}-$sets. Thus f is $(1,2)-\mathcal{C}-$continuous.

Definition 3.2.39. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be $(i,j) - \mathcal{B} - continuous$ if $f^{-1}(V) \in (i,j) - \mathcal{B}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

We can see that if f is an $(i,j)-\mathcal{A}-$continuous, then f is an $(i,j)-\mathcal{B}-$continuous. But the converse is not true.

Example 3.2.40. Let $X = \{1,2,3\}$ and $Y = \{a,b,c\}$.

Consider $m-$structures on X and Y as follows :

$m_X^1 = \{\emptyset, \{1\}, \{2\}, \{2,3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{3\}, \{2,3\}, X\}$.

$m_Y^1 = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, Y\}$ and $m_Y^2 = \{\emptyset, \{a,b\}, \{b,c\}, Y\}$.

Let $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ and $f(\emptyset) = \emptyset, f(\{1\}) = \{a\}, f(\{2\}) = \{b\}$, $f(\{3\}) = \{c\}, f(X) = Y$.

Consider $\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, Y \in m_Y^1$, we get $f^{-1}(\emptyset) = \emptyset, f^{-1}(\{a\}) = \{1\}$, $f^{-1}(\{b\}) = \{2\}, f^{-1}(\{a,b\}) = \{1,2\}, f^{-1}(\{b,c\}) = \{2,3\}, f^{-1}(Y) = X$ are $(1,2)m_X-$ $\mathcal{B}-$sets. Thus f is $(1,2)-\mathcal{B}-$continuous.

Theorem 3.2.41. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. If a mapping $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is $(i,j) - semi-$continuous and $(i,j) - \mathcal{C}-$continuous then f is $(i,j) - \mathcal{A}-$continuous.

Proof. Let f be $(i,j) - semi-$continuous and $(i,j) - \mathcal{C}-$continuous, and let $V \in m_Y^i$. Then $f^{-1}(V) \in (i,j)-SO(X, m_X^1, m_X^2)$ and $f^{-1}(V) \in (i,j)-\mathcal{C}(X, m_X^1, m_X^2)$. By Theorem 3.2.32, thus $f^{-1}(V) \in (i,j)-\mathcal{A}(X, m_X^1, m_X^2)$. Therefore, f is $(i,j)-\mathcal{A}-$continuous. $\square$

CHAPTER 4

$\mathcal{A}-$CONNECTED SETS IN BIMINIMAL STRUCTURE SPACES

In this section, we introduce the concept of $\mathcal{A}-$connected sets in biminimal structure spaces and study some fundamental properties of $\mathcal{A}-$connected sets in biminimal structure spaces.

4.1 $\mathcal{A}-$separated sets in biminimal structure spaces

In this section, we will introduce the notion of $\mathcal{A}-$separated sets in biminimal structure spaces and investigate some of their properties.

Definition 4.1.1. Let (X, m_X^1, m_X^2) be a biminimal structure space and let $M \subseteq X$. Then M is an $(i,j)m_X - \mathcal{A}^C - set$ if $X \setminus M$ is an $(i,j)m_X - \mathcal{A} - set$.

The family of all $(i,j)m_X - \mathcal{A}^C -$sets in a biminimal structure space (X, m_X^1, m_X^2) is denoted by $(i,j) - \mathcal{A}^C\,(X, m_X^1, m_X^2)$.

Example 4.1.2. Let $X = \{1,2,3\}$. Define m-structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$. Thus $(i,j) - \mathcal{A}\,(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$. Then $(i,j) - \mathcal{A}^C\,(X, m_X^1, m_X^2) = \{\emptyset, \{2,3\}, \{1,3\}, \{2\}, X\}$.

Definition 4.1.3. Let (X, m_X^1, m_X^2) be a biminimal structure space and let $M \subseteq X$. Then the $\mathcal{A} - closure$ of M and the $\mathcal{A} - interior$ of M, denoted by $\mathcal{A}cl(M)$ and $\mathcal{A}int(M)$, respectively, are denoted as the following:

$\mathcal{A}cl(M) = \cap\{F : M \subseteq F, F \in (i,j) - \mathcal{A}^C\,(X, m_X^1, m_X^2)\}$;

$\mathcal{A}int(M) = \cup\{G : G \subseteq M, G \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)\}$.

Example 4.1.4. Let $X = \{1,2,3\}$. Define m-structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$. Then $(1,2) - \mathcal{A}(X, m_X^1, m_X^2) = \{\emptyset, \{1,3\}, \{2\}, \{1\}, X\}$ and $(i,j) - \mathcal{A}^C\,(X, m_X^1, m_X^2) = \{\emptyset, \{2\}, \{1,3\}, \{2,3\}, X\}$. Let $M = \{1\} \subseteq X$. Then $\mathcal{A}cl(M) = \{1,3\}$ and $\mathcal{A}int(M) = \{1\}$.

Proposition 4.1.5. Let (X, m_X^1, m_X^2) be a biminimal structure space and $M \subseteq X$. Then $\mathcal{A}cl(X \setminus M) = X \setminus \mathcal{A}int(M)$ and $\mathcal{A}int(X \setminus M) = X \setminus \mathcal{A}cl(M)$.

Proof. Let $M \subseteq X$.

Then $X \setminus \mathcal{A}int(M) = X \setminus \cup\{G : G \subseteq M, G \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)\}$

$= \cap\{X \setminus G : G \subseteq M, G \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)\}$

$= \cap\{X \setminus G : X \setminus M \subseteq X \setminus G, X \setminus G \in (i,j) - \mathcal{A}^C(X, m_X^1, m_X^2)\}$

$= \mathcal{A}cl(X \setminus M)$.

Consequently, we have $\mathcal{A}int(X \setminus M) = X \setminus \mathcal{A}cl(M)$. □

Proposition 4.1.6. Let (X, m_X^1, m_X^2) be a biminimal structure space and $M \subseteq X$. Then

(1) $\mathcal{A}int(M) \subseteq M$;

(2) If $M \subseteq K$, then $\mathcal{A}int(M) \subseteq \mathcal{A}int(K)$;

(3) If M is $(i,j)m_X - \mathcal{A}-$set then $\mathcal{A}int(M) = M$.

Proof. (1) Since $\cup\{G : G \subseteq M, G \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)\} \subseteq M$.

Then $\mathcal{A}int(M) \subseteq M$.

(2) Let $M \subseteq K$, then $\cup\{G : G \subseteq M, G \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)\} \subseteq \cup\{H : H \subseteq K, H \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)\}$. Hence $\mathcal{A}int(M) \subseteq \mathcal{A}int(K)$.

(3) Let M is an $(i,j)m_X - \mathcal{A}-$set. Since $M \subseteq M$ and $M \in \{G : G \subseteq M, G \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)\}$. Then $M \subseteq \cup\{G : G \subseteq M, G \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)\} = \mathcal{A}int(M)$. By (1), $\mathcal{A}int(M) \subseteq M$. Hence $\mathcal{A}int(M) = M$. □

Proposition 4.1.7. Let (X, m_X^1, m_X^2) be a biminimal structure space and $M \subseteq X$.Then

(1) $M \subseteq \mathcal{A}cl(M)$.

(2) If $M \subseteq K$, then $\mathcal{A}cl(M) \subseteq \mathcal{A}cl(K)$;

(3) If M is $(i,j)m_X - \mathcal{A}^C-$set, then $\mathcal{A}cl(M) = M$

Proof. (1) Since $\mathcal{A}int(X - M) \subseteq X \setminus M$. Then $M \subseteq X - \mathcal{A}int(X \setminus M)$.

By Proposition 4.1.5, $M \subseteq \mathcal{A}cl(M)$.

(2) Let $M \subseteq K$, then $\cap\{F : M \subseteq F, F \in (i,j) - \mathcal{A}^C(X, m_X^1, m_X^2)\} \subseteq \cap\{E : K \subseteq E, E \in (i,j) - \mathcal{A}^C(X, m_X^1, m_X^2)\}$. Hence $\mathcal{A}cl(M) \subseteq \mathcal{A}cl(K)$.

(3) Let M is an $(i,j)m_X - \mathcal{A}^C-$set. It follows that $X \setminus M$ is an $(i,j)m_X - \mathcal{A}-$set. By Proposition 4.1.6, $\mathcal{A}int(X \setminus M) = X \setminus M$. By Proposition 4.1.5, $X \setminus \mathcal{A}cl(M) = X \setminus M$. Then $\mathcal{A}cl(M) = M$. □

Proposition 4.1.8. Let (X, m_X^1, m_X^2) be a biminimal structure space and $M \subseteq X$. Then

(1) $x \in \mathcal{A}cl(M)$ if and only if $M \cap V \neq \phi$ for every $(i,j)m_X - \mathcal{A}-$set V

containing x.

(2) $x \in \mathcal{A}int(M)$ if and only if there exists an $(i,j)m_X - \mathcal{A}-$set U such that $U \subseteq M$ and $x \in U$.

Proof. (1) ($\Rightarrow$) Suppose there is an $(i,j)m_X - \mathcal{A}-$set V containing x such that $M \cap V = \emptyset$. Then $X \setminus V$ is an $(i,j)m_X - \mathcal{A}^C-$set such that $M \subseteq X \setminus V$ and $x \notin X \setminus V$. It follows that $x \notin \mathcal{A}cl(M)$.

($\Leftarrow$) Suppose $x \notin \mathcal{A}cl(M)$. Then there exists E such that $M \subseteq E \in (i,j) - \mathcal{A}^C(X, m_X^1, m_X^2)$ but $x \notin E$. It follows that $X \setminus E$ is an $(i,j)m_X - \mathcal{A}-$set containing x such that $M \cap (X \setminus E) = \emptyset$.

(2) It obvious, by Definition 4.1.3. □

Definition 4.1.9. Let (X, m_X^1, m_X^2) be a biminimal structure space and let $Y \subseteq X$ and $M \subseteq Y$. Then an $\mathcal{A}_Y - closure$ of M is defined as follows:

$$\mathcal{A}cl_Y(M) = \mathcal{A}cl(M) \cap Y.$$

Example 4.1.10. Let $X = \{1,2,3\}$ and $Y = \{2,3\}$. Define m-structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$. Let $M = \{3\}$, then $\mathcal{A}cl(M) = \{3\}$. Hence $\mathcal{A}cl_Y(M) = \mathcal{A}cl(M) \cap Y = \{3\} \cap \{2,3\} = \{3\}$. Thus $\mathcal{A}cl(M) = \{3\}$. Then $\mathcal{A}cl_Y(M) = \{3\}$.

Definition 4.1.11. Let (X, m_X^1, m_X^2) be a biminimal structure space and let $K, M \subseteq X$. Then K and M are $(i,j)\mathcal{A} - separated$ if and only if $\mathcal{A}cl(K) \cap M = \emptyset = \mathcal{A}cl(M) \cap K$, where $(i,j) = 1,2$ and $i \neq j$.

Moreover, if (Y, m_Y^1, m_Y^2) be a biminimal subspace of X, then $U, V \subseteq Y$ be $(i,j)\mathcal{A}-$separated in Y if $\mathcal{A}cl_Y(U) \cap V = \emptyset$ and $\mathcal{A}cl_Y(V) \cap U = \emptyset$.

Example 4.1.12. Let $X = \{1,2,3\}$. Define m-structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$. Thus $(1,2) - \mathcal{A}(X, m_X^1, m_X^2) = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$. Then $(1,2) - \mathcal{A}^C(X, m_X^1, m_X^2) = \{\emptyset, \{2,3\}, \{1,3\}, \{2\}, X\}$. Let $K = \{1\}$ and $M = \{2\}$. It follows that $\mathcal{A}cl(K) = \{1,3\}$ and $\mathcal{A}cl(M) = \{2\}$. Thus $\mathcal{A}cl(K) \cap M = \emptyset$ and $\mathcal{A}cl(M) \cap K = \emptyset$. Therefore, K and M are $(1,2)\mathcal{A}-$separated.

Theorem 4.1.13. Let (X, m_X^1, m_X^2) be a biminimal structure space and (Y, m_Y^1, m_Y^2) be a biminimal subspace of X and let $U, V \subseteq Y$. Then U, V be $(i,j)\mathcal{A}-$separated in X iff U and V be $(i,j)\mathcal{A}-$separated in Y.

Proof. ($\Rightarrow$) Let U and V be $(i,j)\mathcal{A}-$separated in X. Then $\mathcal{A}cl(U) \cap V = \emptyset$ and $\mathcal{A}cl(V) \cap U = \emptyset$, so $(\mathcal{A}cl(U) \cap Y) \cap V = \emptyset$ and $(\mathcal{A}cl(V) \cap Y) \cap U = \emptyset$. Thus $\mathcal{A}cl_Y(U) \cap V = \emptyset$ and $\mathcal{A}cl_Y(V) \cap U = \emptyset$. Hence U and V be $(i,j)\mathcal{A}-$separated in Y.

($\Leftarrow$) Let U and V be $(i,j)\mathcal{A}-$separated in Y. Then $\mathcal{A}cl_Y(U) \cap V = \emptyset$ and $\mathcal{A}cl_Y(V) \cap U = \emptyset$. Thus $(\mathcal{A}cl(U) \cap Y) \cap V = \emptyset$ and $(\mathcal{A}cl(V) \cap Y) \cap U = \emptyset$. Since $U, V \subseteq Y$ so $\mathcal{A}cl(U) \cap V = \emptyset$ and $\mathcal{A}cl(V) \cap U = \emptyset$. Hence U and V be $(i,j)\mathcal{A}-$separated in X. □

Proposition 4.1.14. Let (X, m_X^1, m_X^2) be a biminimal structure space and $K, M \subseteq X$. If K and M are $(i,j)\mathcal{A}-$separated then K and M are disjoint.

Proof. Let K and M be $(i,j)\mathcal{A}-$separated. Then $\mathcal{A}cl(K) \cap M = \emptyset = \mathcal{A}cl(M) \cap K$. By Proposition 4.1.7, $K \subseteq \mathcal{A}cl(K)$ and $M \subseteq \mathcal{A}cl(M)$. Then $K \cap M = \emptyset$. Thus K and M are disjoint. □

Remark. Let (X, m_X^1, m_X^2) be a biminimal structure space and $K, M \subseteq X$. By Proposition 4.1.14, if K and M are $(i,j) - \mathcal{A}-$separated then K and M are disjoint. But the converse is not true, i.e. if K and M are disjoint, then K and M does not need be $(i,j)\mathcal{A}-$separated as can be seen from the following example.

Example 4.1.15. Let $X = \{1, 2, 3\}$. Define $m-$structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, X\}$. Let $K = \{1, 2\}$ and $M = \{3\}$. Then $\mathcal{A}cl(K) = X$ and $\mathcal{A}cl(M) = \{1, 3\}$. Thus $\mathcal{A}cl(K) \cap M = \{3\} \neq \emptyset$ and $\mathcal{A}cl(M) \cap K = \{1\} \neq \emptyset$. Hence K and M are not $(i,j)\mathcal{A}-$separated.

Proposition 4.1.16. Let (X, m_X^1, m_X^2) be a biminimal structure space and $K, M \subseteq X$. If K and M are $(i,j)\mathcal{A}-$separated, then D and E are $(i,j)\mathcal{A}-$separated, where $D \subseteq K$ and $E \subseteq M$.

Proof. Let K and M are $(i,j)\mathcal{A}-$separated. Then $\mathcal{A}cl(K) \cap M = \emptyset = \mathcal{A}cl(M) \cap K$. Since $D \subseteq K$ and $E \subseteq M$. Then $\mathcal{A}cl(D) \cap E = \emptyset = \mathcal{A}cl(E) \cap D$. Therefore, D and E are $(i,j)\mathcal{A}-$separated. □

Definition 4.1.17. Let (X, m_X^1, m_X^2) be a biminimal structure space. Then (X, m_X^1, m_X^2) is said to be a $T_{\mathcal{A}}-space$ if the arbitary union of $(i,j)m_X-\mathcal{A}-$sets is an $(i,j)m_X-\mathcal{A}-$set.

Example 4.1.18. Let $X = \{1,2,3\}$. Define m-structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1,2\}, \{1,3\}, X\}$ and $m_X^2 = \{\emptyset, \{2\}, \{1,2\}, X\}$. Then $(1,2){-}\mathcal{A}(X, m_X^1, m_X^2)$ $= \{\emptyset, \{1,2\}, \{1,3\}, X\}$. Thus $(1,2) - \mathcal{A}(X, m_X^1, m_X^2)$ is $T_{\mathcal{A}}-$space.

Remark. By Definition 4.1.17, if (X, m_X^1, m_X^2) is a $T_{\mathcal{A}}-$space, then every intersection of $(i,j)m_X - \mathcal{A}^C-$sets is $(i,j)m_X - \mathcal{A}^C-$sets as well.

Proposition 4.1.19. Let (X, m_X^1, m_X^2) be a biminimal structure space and $K, M \subseteq X$. If (X, m_X^1, m_X^2) is a $T_{\mathcal{A}}-$space, then the following statements are equivalent:

(1) K and M are $(i,j)\mathcal{A}-$separated.

(2) There are $(i,j)m_X - \mathcal{A}^C-$sets F_K and F_M such that $K \subseteq F_K \subseteq (X \setminus M)$ and $M \subseteq F_M \subseteq (X \setminus K)$;

(3) There are $(i,j)m_X - \mathcal{A}-$sets G_K and G_M such that $K \subseteq G_K \subseteq (X \setminus M)$ and $M \subseteq G_M \subseteq (X \setminus K)$.

Proof. (1)⇒(2) Let K and M are $(i,j)\mathcal{A}-$separated. Then $\mathcal{A}cl(K) \cap M = \emptyset = \mathcal{A}cl(M) \cap K$. Since (X, m_X^1, m_X^2) is $T_{\mathcal{A}}-$ space, $\mathcal{A}cl(K)$ and $\mathcal{A}cl(M)$ are $(i,j)m_X - \mathcal{A}^C-$sets. It follows that $K \subseteq \mathcal{A}cl(K) \subseteq (X \setminus M)$ and $M \subseteq \mathcal{A}cl(M) \subseteq (X \setminus K)$.

(2)⇒(1) Let $K \subseteq F_K \subseteq (X \setminus M)$ and $M \subseteq F_M \subseteq (X \setminus K)$ for some $F_K, F_M \in (i,j) - \mathcal{A}^C(X, m_X^1, m_X^2)$. It follows that $F_K \cap M = \emptyset = F_M \cap K$. Since $K \subseteq F_K$ and $M \subseteq F_M$, $\mathcal{A}cl(K) \subseteq \mathcal{A}cl(F_K)$ and $\mathcal{A}cl(M) \subseteq \mathcal{A}cl(F_M)$. By Proposition 4.1.16, $\mathcal{A}cl(F_K) = F_K$ and $\mathcal{A}cl(F_M) = F_M$ and $\mathcal{A}cl(K) \subseteq F_K$ and $\mathcal{A}cl(M) \subseteq F_M$. Thus $\mathcal{A}cl(K) \cap M = \emptyset = \mathcal{A}cl(M) \cap K$. Therefore, K and M are $(i,j)\mathcal{A}-$separated.

(2)⇒(3) Suppose that $K \subseteq F_K \subseteq (X \setminus M)$ and $M \subseteq F_M \subseteq (X \setminus K)$ for some $F_K, F_M \in (i,j) - \mathcal{A}^C(X, m_X^1, m_X^2)$. Hence $X \setminus F_K$ and $X \setminus F_M$ are $(i,j)m_X - \mathcal{A}-$sets. Thus $M \subseteq (X \setminus F_K) \subseteq (X \setminus K)$ and $K \subseteq (X \setminus F_M) \subseteq (X \setminus M)$. Set $G_K = X \setminus F_M$ and $G_M = X \setminus F_K$. Therefore, $K \subseteq G_K \subseteq (X \setminus M)$ and $M \subseteq G_M \subseteq (X \setminus K)$.

(3)⇒(2) Suppose that $K \subseteq G_K \subseteq (X \setminus M)$ and $M \subseteq G_M \subseteq (X \setminus K)$ for some $G_K, G_M \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)$. Hence $X \setminus G_K$ and $X \setminus G_M$ are $(i,j)m_X - \mathcal{A}^C-$sets. Thus $M \subseteq (X \setminus G_K) \subseteq (X \setminus K)$ and $K \subseteq (X \setminus G_M) \subseteq (X \setminus M)$. Set $F_K = X \setminus G_M$ and $F_M = X \setminus G_K$. Therefore, $K \subseteq F_K \subseteq (X \setminus M)$ and $M \subseteq F_M \subseteq (X \setminus K)$.

4.2 $\mathcal{A}-$connected sets in biminimal structure spaces

In this section, we will introduce the notion of $\mathcal{A}-$connected sets in biminimal structure spaces and investigate some of their properties.

Definition 4.2.1. Let C be a nonempty subset of a biminimal structure space (X, m_X^1, m_X^2). Then C is an $(i,j)\mathcal{A}-$*connected set* of X if and only if for any two subsets K and M such that $C = K \cup M$, K and M are $(i,j)\mathcal{A}-$separated sets imply either $K = \emptyset$ or $M = \emptyset$. The space X is said to be an $(i,j)\mathcal{A}-$connected set iff it is an $(i,j)\mathcal{A}-$connected subset of itself, where $(i,j) = 1, 2$ and $i \neq j$.

Example 4.2.2. Let $X = \{1,2,3\}$. Define m-structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$. We have $(1,2)-\mathcal{A}\,(X, m_X^1, m_X^2)$ $= \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$ and $(1,2) - \mathcal{A}^C\,(X, m_X^1, m_X^2) = \{\emptyset, \{2,3\}, \{1,3\}, \{2\}, X\}$. Let $C = \{1,2\} \subseteq X$. We can see that $\{1\}$and $\{2\}$ are $(1,2)\mathcal{A}-$separated such that $C = \{1\} \cup \{2\}$ but $\{1\} \neq \emptyset \neq \{2\}$. It follows that C is not $(i,j)\mathcal{A}-$connected in (X, m_X^1, m_X^2).

Consider $\{1,3\} \subseteq X$. We can see that for every subset M and K. Such that $\{1,3\} = K \cup M$, K and M are $(1,2)\mathcal{A}-$separated imply $K = \emptyset$ or $M = \emptyset$.

Proposition 4.2.3. Let (X, m_X^1, m_X^2) be a biminimal structure space and (X, m_X^1, m_X^2) is a $T_{\mathcal{A}}-$space, then the following statements are equivalent:

(1) The space X is $(i,j)\mathcal{A}-$connected sets;

(2) If $X = G_1 \cup G_2, G_1 \cap G_2 = \emptyset, G_1$ and G_2 are $(i,j)m_X - \mathcal{A}-$set then either $G_1 = \emptyset$ or $G_2 = \emptyset$;

(3) If $X = F_1 \cup F_2, F_1 \cap F_2 = \emptyset, F_1$ and F_2 are $(i,j)m_X - \mathcal{A}^C-$set, then either $F_1 = \emptyset$ or $F_2 = \emptyset$;

(4) If $H \subseteq X$ is both $(i,j)m_X - \mathcal{A}-$set and $(i,j)m_X - \mathcal{A}^C-$set, then either $H = \emptyset$ or $H = X$.

Proof. (1)⇒(2) Assume that X is $(i,j)\mathcal{A}-$connected. Let $X = G_1 \cup G_2, G_1 \cap G_2 = \emptyset$ and $G_1, G_2 \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)$. Then G_1 and G_2 are $(i,j)m_X - \mathcal{A}-$sets such that $G_1 \subseteq G_1 \subseteq (X \setminus G_2)$ and $G_2 \subseteq G_2 \subseteq (X \setminus G_1)$. By Proposition 4.1.19, G_1 and G_2 are $(i,j)\mathcal{A}-$separated sets. Since X is $(i,j)\mathcal{A}-$connected and $X \neq \emptyset$ thus either $G_1 = \emptyset$ or

$G_2 = \emptyset$.

(2)⇒(3) Let $X = F_1 \cup F_2, F_1 \cap F_2 = \emptyset$ and $F_1, F_2 \in (i,j) - \mathcal{A}^C(X, m_X^1, m_X^2)$. Set $G_1 = X \setminus F_1$ and $G_2 = X \setminus F_2$. It follows that $G_1, G_2 \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)$. Since $G_1 \cup G_2 = (X \setminus F_1) \cup (X \setminus F_2) = X \setminus (F_1 \cap F_2) = (X \setminus \emptyset) = X$ and $G_1 \cap G_2 = (X \setminus F_1) \cap (X \setminus F_2) = X \setminus (F_1 \cup F_2) = X \setminus X = \emptyset$. By the assumption, either $X \setminus F_2 = G_2 = \emptyset$ or $X \setminus F_1 = G_1 = \emptyset$. By the assumption, either $F_1 = \emptyset$ or $F_2 = \emptyset$.

(3)⇒(4) Let $H \subseteq X$ and H be both an $(i,j)m_X - \mathcal{A}-$set and $(i,j)m_X - \mathcal{A}^C-$set. Then $X \setminus H$ is both an $(i,j)m_X - \mathcal{A}-$set and $(i,j)m_X - \mathcal{A}^C-$set. Since $X = H \cup (X \setminus H)$, $H \cap (X \setminus H) = \emptyset$ and $H, X \setminus H \in (i,j)m_X - \mathcal{A}^C(X, m_X^1, m_X^2)$. By the assumption, either $H = \emptyset$ or $X \setminus H = \emptyset$. Hence either $H = \emptyset$ or $H = X$.

(4)⇒(2) Let $X = G_1 \cup G_2, G_1 \cap G_2 = \emptyset$ and $G_1, G_2 \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)$. Since $G_1 = X \setminus G_2$, G_1 is an $(i,j)m_X - \mathcal{A}^C-$set. By the assumption, either $G_1 = \emptyset$ or $G_1 = X$. Hence either $G_1 = \emptyset$ or $G_2 = \emptyset$.

(2)⇒(1) Let $X = K \cup M$ and K, M are $(i,j)\mathcal{A}-$separated. Set $G_1 = X \setminus \mathcal{A}cl(K)$ and $G_2 = X \setminus \mathcal{A}cl(M)$. Since X is $T_{\mathcal{A}}-$space, G_1 and G_2 are $(i,j)m_X - \mathcal{A}-$set. Then $M \subseteq X \setminus \mathcal{A}cl(K)$ and $K \subseteq X \setminus \mathcal{A}cl(M)$. Thus $M \subseteq G_1$ and $K \subseteq G_2$. Hence $G_1 = M$ and $G_2 = K, G_1 \cap G_2 = \emptyset$. Therefore, $G_1 = M = \emptyset$ and $G_2 = K = \emptyset$. □

Lemma 4.2.4. Let (X, m_X^1, m_X^2) be a biminimal structure space and $K, M \subseteq X$. If C is an $(i,j)\mathcal{A}-$connected $C \subseteq K \cup M$, K and M are $(i,j)\mathcal{A}-$separated, then either $C \subseteq K$ or $C \subseteq V$.

Proof. Let C be an $(i,j)\mathcal{A}-$connected, $C \subseteq K \cup M$,K and M be $(i,j)\mathcal{A}-$separated. Then $C = C \cap (K \cup M) = (C \cap K) \cup (C \cap M)$. Since K and M are $(i,j)\mathcal{A}-$separated. Then $\mathcal{A}cl(K) \cap M = \emptyset = \mathcal{A}cl(M) \cap K$. Since $\mathcal{A}cl(C \cap K) \subseteq \mathcal{A}cl(K)$ and $C \cap M \subseteq M$. Hence $\mathcal{A}cl(C \cap K) \cap (C \cap M) \subseteq \mathcal{A}cl(K) \cap M = \emptyset$. Similary $\mathcal{A}cl(C \cap M) \cap (C \cap K) = \emptyset$. Consequently $(C \cap K)$ and $(C \cap M)$ are $(i,j)\mathcal{A}-$separated. Since $C = (C \cap K) \cup (C \cap M)$ is $(i,j)\mathcal{A}-$connected, either $C \cap K = \emptyset$ or $C \cap M = \emptyset$. It follows that either $C = \emptyset \cup (C \cap M)$ or $S = (C \cap K) \cup \emptyset$. Hence either $C \subseteq M$ or $C \subseteq K$. □

Theorem 4.2.5. Let (X, m_X^1, m_X^2) be a biminimal structure space.
If C is an $(i,j)\mathcal{A}-$connected set, $C \subseteq B \subseteq \mathcal{A}cl(C)$then C is an $(i,j)\mathcal{A}-$connected set.

Proof. Let $B = K \cup M$, K and M be $(i,j)\mathcal{A}-$separated. Consequently $\mathcal{A}cl(K) \cap M = \emptyset$

and $\mathcal{A}cl(M) \cap K = \emptyset$. It follows that $\mathcal{A}cl(K) \subseteq (X \setminus M)$ and $\mathcal{A}cl(M) \subseteq (X \setminus K)$. Since $C \subseteq B = K \cup M$ and by Lemma 4.2.4, either $C \subseteq K$ or $C \subseteq M$. So either $B \subseteq \mathcal{A}cl(C) \subseteq \mathcal{A}cl(K) \subseteq (X \setminus M)$ or $B \subseteq \mathcal{A}cl(C) \subseteq \mathcal{A}cl(M) \subseteq (X \setminus K)$. Therefore, either $M = \emptyset$ or $K = \emptyset$. □

Corollary 4.2.6. Let (X, m_X^1, m_X^2) be a biminimal structure space.
If C is $(i,j)\mathcal{A}-$connected sets, then $\mathcal{A}cl(C)$ is $(i,j)\mathcal{A}-$connected sets.

Lemma 4.2.7. Let (X, m_X^1, m_X^2) be a biminimal structure space.
If C_α is $(i,j)\mathcal{A}-$connected for all $\alpha \in J$ and for $\beta, \gamma \in J$, $\beta \neq \gamma$, C_β and C_γ are not $(i,j)\mathcal{A}-$separated, then $\bigcup_{\alpha \in J} C_\alpha$ is $(i,j)\mathcal{A}-$connected as well.

Proof. Let $\bigcup_{\alpha \in J} C_\alpha = K \cup M$, K and M are $(i,j)\mathcal{A}-$separated. It follows that $K \cap M = \emptyset$. Since $C_\alpha \subseteq \bigcup_{\alpha \in J} C_\alpha$ and by Lemma 4.2.4, either $C_\alpha \subseteq K$ or $C_\alpha \subseteq M$ for all $\alpha \in J$. Since C_β and C_γ are not $(i,j)\mathcal{A}-$separated for all $\beta, \gamma \in J$ and $\beta \neq \gamma$, then there does not exist $\beta, \gamma \in J$ such that $C_\beta \subseteq K$ and $C_\gamma \subseteq M$. Then either $C_\alpha \subseteq K$, $\forall \alpha \in J$ or $C_\alpha \subseteq M$, $\forall \alpha \in J$. In the first case $\bigcup_{\alpha \in J} C_\alpha \subseteq K$ and $M = \emptyset$. In the second one $\bigcup_{\alpha \in J} C_\alpha \subseteq M$ and $K = \emptyset$. □

Corollary 4.2.8. Let (X, m_X^1, m_X^2) be a biminimal structure space and $C = \bigcup_{\alpha \in J} C_\alpha$. If C_α is $(i,j)\mathcal{A}-$connected for all $\alpha \in J$ and $C_\beta \cap C_\gamma \neq \emptyset$ for all $\beta, \gamma \in J$ then C is $(i,j)\mathcal{A}-$connected.

Corollary 4.2.9. Let (X, m_X^1, m_X^2) be a biminimal structure spaces and $C = \bigcup_{\alpha \in J} C_\alpha$. If C_α is an $(i,j)\mathcal{A}-$connected for all $\alpha \in J$ and $\bigcap_{\alpha \in J} C_\alpha \neq \emptyset$ then C is an $(i,j)\mathcal{A}-$connected.

Definition 4.2.10. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. Let $f : X \to Y$, we will say that f is $(i,j)(\mathcal{A}_X, \mathcal{A}_Y)-$continuous iff $f^{-1}(W) \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)$ for all $W \in (i,j) - \mathcal{A}(Y, m_Y^1, m_Y^2)$.

Remark. By Definition 4.2.10, if f is $(i,j)(\mathcal{A}_X, \mathcal{A}_Y)-$continuous, then f is $(i,j) - \mathcal{A}-$continuous as well.

Example 4.2.11. Let $X = \{1,2,3\} = Y$. Consider $m-$structures on X and Y as follows: $m_X^1 = \{\emptyset, \{1\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1,3\}, X\}$.

$m_Y^1 = \{\emptyset, \{1\}, \{2\}, \{2,3\}, Y\}$ and $m_Y^2 = \{\emptyset, \{1\}, \{3\}, \{2,3\}, Y\}$.

By definition 4.2.10, consider $\emptyset, \{1\}, \{2\}, \{2,3\} \in (1,2)-\mathcal{A}(Y, m_Y^1, m_Y^2)$, we get $f^{-1}(\emptyset) = \emptyset, f^{-1}(\{1\}) = \{1,3\}, f^{-1}(\{2\}) = \{1\}, f^{-1}(\{2,3\}) = \{2\}$ are $(1,2)m_X - \mathcal{A}-$sets. Thus f is $(\mathcal{A}_X, \mathcal{A}_Y)-$ continuous.

Lemma 4.2.12. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be $T_{\mathcal{A}}-$ spaces.
If f is $(i,j)(\mathcal{A}_X, \mathcal{A}_Y)-$continuous and $V, W \subseteq Y$ are $(i,j)\mathcal{A}-$separated then $f^{-1}(V)$ and $f^{-1}(W)$ are $(i,j)\mathcal{A}-$separated.

Proof. By Proposition 4.1.19, there exist G_V and G_W are $(i,j)m_Y-\mathcal{A}-$sets such that $V \subseteq G_V \subseteq (Y \setminus W)$ and $W \subseteq G_W \subseteq (Y \setminus V)$. Then $f^{-1}(V) \subseteq f^{-1}(G_V) \subseteq (X \setminus f^{-1}(W))$ and $f^{-1}(W) \subseteq f^{-1}(G_W) \subseteq (X \setminus f^{-1}(V))$, with $f^{-1}(G_V)$ and $f^{-1}(G_W)$ are $(i,j)m_X - \mathcal{A}-$sets. By Proposition 4.1.19, $f^{-1}(V)$ and $f^{-1}(W)$ are $(i,j)\mathcal{A}-$separated. □

Theorem 4.2.13. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be $T_{\mathcal{A}}-$spaces. If $C \subseteq X$ is $(i,j)\mathcal{A}-$connected and f is $(i,j)(\mathcal{A}_X, \mathcal{A}_Y)-$ continuous then $f(C)$ is $(i,j)\mathcal{A}-$connected.

Proof. Suppose $f(C) = V \cup W$, V and W be $(i,j)\mathcal{A}-$separated. Since $f(C) = V \cup W$, then $C \subseteq (f^{-1}(V) \cup f^{-1}(W))$. By the hypothesis $f^{-1}(V)$ and $f^{-1}(W)$ are $(i,j)\mathcal{A}-$sets. By Lemma 4.2.12, $f^{-1}(V)$ and $f^{-1}(W)$ are $(i,j)\mathcal{A}-$separated. By Lemma 4.2.4, either $C \subseteq f^{-1}(V)$ or $C \subseteq f^{-1}(W)$, i.e either $f(C) \subseteq V$ or $f(C) \subseteq W$. It follows that $W = \emptyset$ or $V = \emptyset$. Hence $f(C)$ is $(i,j)\mathcal{A}-$connected. □

CHAPTER 5

CONCLUSIONS

The aim of this thesis is to introduce the concepts of $\mathcal{A}-$sets in biminimal structure spaces. And we study some properties of $(i,j)\mathcal{A}-$continuous on the space. Moreover, we introduce the concepts of some $\mathcal{A}-$connected by using $\mathcal{A}-$separated and study relationships other types of $\mathcal{A}-$connected on biminimal structure spaces and study some of their properties. The results are follows:

1) Let (X, m_X) be an $m-$space. A subset M of X is said to be an $m_X - \mathcal{A}-$set if there exist G and R such that $M = G \cap R$ when G is open and R is a m_X-regular closed.

2) Let (X, m_X) be an $m-$space and $A \subseteq X$, then A is said to be an $m_X - t-$set if $m_X Int(A) = m_X Int(m_X Cl(A))$.

3) Let (X, m_X) be an $m-$space and $R \subseteq X$. If R is m_X- regular closed then R is $m_X - t-$set.

4) A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be $(i,j)m_X-$locally closed if there exist G and F such that $A = G \cap F$ when G is an m_X^i-open set G and F is an m_X^j-closed set, where $i, j = 1, 2$ and $i \neq j$.

   From the above definitions, I have the following theorems are derived:

   4.1) Let S be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and let $i, j = 1, 2$ and $i \neq j$. If S is an $(i,j)m_X-$locally closed set then there exists an m_X^i-open set U such that $S = U \cap m_X^j Cl(S)$.

   4.2) Let S be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and let m_X^j has property $\mathfrak{B}$, where $i, j = 1, 2$ and $i \neq j$. Then S is an $(i,j)m_X-$locally closed set iff there exists an m_X^i-open set U such that $S = U \cap m_X^j Cl(S)$.

   4.3) Let (X, m_X^1, m_X^2) be a biminimal structure space m_X^j has the property $\mathfrak{B}$. If a subset M of X is an $(i,j)m_X-\mathcal{A}-$set, then M is $(i,j)m_X-$locally closed, where $i, j = 1, 2$ and $i \neq j$.

4.4) Let (X, m_X^1, m_X^2) be a biminimal structure space and $m_X^j \subseteq m_X^i$ has the property $\mathfrak{B}$. If a subset M of X is both $(i,j)m_X-$semi$-$open and $(i,j)m_X-$locally closed, then M is an $(i,j)m_X - \mathcal{A}-$set, where $i, j = 1, 2$ and $i \neq j$.

4.5) Let (X, m_X^1, m_X^2) be a biminimal structure space and M be a subset of X. If m_X^j has the property $\mathfrak{B}$ and M is an $(i,j)m_X-$locally closed set, then it is also an $(i,j)m_X - \mathcal{B}-$set, where $i, j = 1, 2$ and $i \neq j$.

4.6) Let (X, m_X^1, m_X^2) be a biminimal structure space and M be a subset of X. If m_X^j has the property $\mathfrak{B}$ and M is an $(i,j)m_X-$locally closed set, then it is also an $(i,j)m_X - \mathcal{C}-$set, where $i, j = 1, 2$ and $i \neq j$.

5) Let (X, m_X^1, m_X^2) be a biminimal structure space. A subset M of X is said to be an $(i,j)m_X - \mathcal{A}-$set if there exists G and R, such that $M = G \cap R$ when $G \in m_X^i$ and R is m_X^j-regular closed, where $i, j = 1, 2$ and $i \neq j$.

From the above definitions, I have the following theorems are derived:

5.1) The intersection of two $(i,j)m_X - \mathcal{A}-$sets may not be an $(i,j)m_X - \mathcal{A}-$set.

5.2) The union of two $(i,j)m_X - \mathcal{A}-$sets may not be an $(i,j)m_X - \mathcal{A}-$set.

5.3) Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$.
If A is an $(i,j)m_X - \mathcal{A}-$set, then A is an $(i,j)m_X - \mathcal{B}-$set for all i, j= 1, 2 and $i \neq j$.

5.4) Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$.
If A is an $(i,j)m_X - \mathcal{A}-$set then it is an $(i,j)m_X - \mathcal{C}-$set for all i, j = 1, 2 and $i \neq j$.

6) Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$.
Then A is said to be an $(i,j)m_X - t-$set if $m_X^i Int(A) = m_X^i Int(m_X^j Cl(A))$, where $i, j = 1, 2$ and $i \neq j$.

From the above definitions, I have the following theorems are derived:

6.1) Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$.
Then A is an $(i,j)m_X - t-$set if and only if A is $(i,j)m_X-$semi$-$closed .

7) Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$.
Then A is said to be an $(i,j)m_X - \mathcal{B}-$set if $A = U \cap T,$ when U is an m_X^i-open set and T is an $m_X^j - t-$set, where $i, j = 1, 2$ and $i \neq j$.

8) Let (X, m_X^1, m_X^2) be a biminimal structure space and $A \subseteq X$.
Then A is said to be $(i,j)m_X - \mathcal{C}-$set if $A = U \cap B,$ when U is an m_X^i-open and B is m_X^j- preclosed, where $i, j = 1, 2$ and $i \neq j$.

From the above definitions, I have the following theorems are derived:

8.1) Let A be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and m_X^j has the property $\mathfrak{B}$. Then A is an $(i,j) - \mathcal{C}-$set iff $A = U \cap m_X^j pcl(A)$ for some $U \in m_X^i$, where $i, j = 1, 2$ and $i \neq j$.

8.2) Let (X, m_X^1, m_X^2) be a biminimal structure space and m_X^j has the property $\mathfrak{B}$. If a subset M of X is an m_X^j-semi$-$open set and $(i,j)m_X - \mathcal{C}-$set, then it is an $(i,j)m_X - \mathcal{A}-$set.

9) Let A be a subset of a biminimal stucture space (X, m_X^1, m_X^2) and m_X^j has the property $\mathfrak{B}$. Then $A = U \cap m_X^j Cl(m_X^j Int(A))$ for some $U \in m_X^i$ if and only if A is an m_X^j-semi$-$open set and $(i,j)m_X - \mathcal{C}-$set.

10) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is said to be

(1) an $(i,j) - semi-$continuous if $f^{-1}(V) \in (i,j) - SO(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

(2) an $(i,j) - \mathcal{LC}-$continuous if $f^{-1}(V) \in (i,j) - \mathcal{LC}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

(3) an $(i,j) - \mathcal{A}-$continuous if $f^{-1}(V) \in (i,j) - \mathcal{A}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

From the above definitions, I have the following theorems are derived:

10.1) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces and let $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ be a mapping. If f is $(i,j) - \mathcal{A}-$continuous then f is $(i,j) - \mathcal{LC}-$continuous.

10.2) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces and m_X^j has the property $\mathfrak{B}$. If a mapping $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is $(i, j)-semi-$continuous and $(i, j)-\mathcal{LC}-$continuous then f is $(i, j)-$ $\mathcal{A}-$continuous.

11) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function f : $(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is said to be $(i, j)-\mathcal{C}-$continuous if $f^{-1}(V) \in (i, j)-$ $\mathcal{C}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

From the above definitions, I have the following theorems are derived:

11.1) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. If a mapping $f : (X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is an $(i, j)-semi-$continuous and $(i, j) - \mathcal{C}-$continuous then f is $(i, j) - \mathcal{A}-$continuous.

12) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function f : $(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is said to be $(i, j) - \mathcal{B}-$continuous if $f^{-1}(V) \in$ $(i, j) - \mathcal{B}(X, m_X^1, m_X^2)$ for all $V \in m_Y^i$.

13) Let (X, m_X^1, m_X^2) be a biminimal structure space and let $M \subseteq X$. Then M is an $(i, j)m_X - \mathcal{A}^C-$set if $X \setminus A$ is an $(i, j)m_X - \mathcal{A}-$set.

14) Let (X, m_X^1, m_X^2) be a biminimal structure space and let $M \subseteq X$. Then the $\mathcal{A}-$closure of M and the $\mathcal{A}-$interior of M, denoted by $\mathcal{A}cl(M)$ and $\mathcal{A}int(M)$, respectively, are denoted as the following :

$\mathcal{A}cl(M) = \cap\{F : M \subseteq F, F \in (i, j) - \mathcal{A}^C\,(X, m_X^1, m_X^2)\}$.

$\mathcal{A}int(M) = \cup\{G : G \subseteq M, G \in (i, j) - \mathcal{A}(X, m_X^1, m_X^2)\}$.

From the above definitions, I have the following theorems are derived:

14.1) Let (X, m_X^1, m_X^2) be a biminimal structure space and $M \subseteq X$.
Then $\mathcal{A}cl(X \setminus M) = X \setminus \mathcal{A}int(M)$ and $\mathcal{A}int(X \setminus M) = X \setminus \mathcal{A}cl(M)$

14.2) Let (X, m_X^1, m_X^2) be a biminimal structure space and $M \subseteq X$.
Then (1) $\mathcal{A}int(M) \subseteq M$.
(2) If $M \subseteq K$, then $\mathcal{A}int(M) \subseteq \mathcal{A}int(K)$.
(3) If M is $(i, j)m_X - \mathcal{A}-$set then $\mathcal{A}int(M) = M$.

15) Let (X, m^1_X, m^2_X) be a biminimal structure space and $M \subseteq X$.
Then (1) $x \in \mathcal{A}cl(M)$ if and only if $M \cap V \neq \phi$ for every $(i,j)m_X - \mathcal{A}-$set V containing x

16) Let (X, m^1_X, m^2_X) be a biminimal structure space and let $Y \subseteq X$ and $M \subseteq Y$. Then an $\mathcal{A}_Y-$closure of M is defined as follows : $\mathcal{A}cl_Y(M) = \mathcal{A}cl(M) \cap Y$.

17) Let (X, m^1_X, m^2_X) be a biminimal structure space and let $K, M \subseteq X$. Then K and M are $(i,j)\mathcal{A}-$separated if and only if $\mathcal{A}cl(K) \cap M = \emptyset = \mathcal{A}cl(M) \cap K$, where $(i,j) = 1,2$ and $i \neq j$.

From the above definitions, I have the following theorems are derived:

17.1) Let (X, m^1_X, m^2_X) be a biminimal structure space and (Y, m^1_Y, m^2_Y) be a biminimal subspace of X and let $U, V \subseteq Y$. Then U, V be $(i,j)\mathcal{A}-$separated in X iff U and V be $(i,j)\mathcal{A}-$separated in Y.

17.2) Let (X, m^1_X, m^2_X) be a biminimal structure space and $K, M \subseteq X$.
If K and M are $(i,j)\mathcal{A}-$separated then K and M are disjoint.

17.3) Let (X, m^1_X, m^2_X) be a biminimal structure space and $K, M \subseteq X$.
If K and M are $(i,j)\mathcal{A}-$separated, then D and E are $(i,j)\mathcal{A}-$separated, where $D \subseteq K$ and $E \subseteq M$.

18) Let (X, m^1_X, m^2_X) be a biminimal structure space. Then (X, m^1_X, m^2_X) is said to be a $T_{\mathcal{A}}-$space if the arbitary union of $(i,j)m_X - \mathcal{A}-$sets is an $(i,j)m_X - \mathcal{A}-$set.

19) Let (X, m^1_X, m^2_X) be a biminimal structure space and $K, M \subseteq X$.
If (X, m^1_X, m^2_X) is a $T_{\mathcal{A}}-$space, then the following statements are equivalent:
(1) K and M are $(i,j)\mathcal{A}-$separated.
(2) There are $(i,j)m_X - \mathcal{A}^C-$sets F_K and F_M such that $K \subseteq F_K \subseteq (X \setminus M)$ and $M \subseteq F_M \subseteq (X \setminus K)$;
(3) There are $(i,j)m_X - \mathcal{A}-$sets G_K and G_M such that $K \subseteq G_K \subseteq (X \setminus M)$ and $M \subseteq G_M \subseteq (X \setminus K)$.

20) Let C be a nonempty subset of a biminimal structure space (X, m^1_X, m^2_X). Then C is an $(i,j)\mathcal{A}-$connected set of X if and only if for any two subsets K and M such that

$C = K \cup M$, K and M are $(i,j)\mathcal{A}-$separated sets imply either $K = \emptyset$ or $M = \emptyset$. The space X is said to be an $(i,j)\mathcal{A}-$connected set iff it is an $(i,j)\mathcal{A}-$connected subset of itself, where $(i,j) = 1,2$ and $i \neq j$.

From the above definitions, I have the following theorems are derived:

20.1) Let (X, m_X^1, m_X^2) be a biminimal structure space and $K, M \subseteq X$. If C is an $(i,j)\mathcal{A}-$connected $C \subseteq K \cup M$, K and M are $(i,j)\mathcal{A}-$separated, then either $C \subseteq K$ or $C \subseteq V$.

20.2) Let (X, m_X^1, m_X^2) be a biminimal structure space.
If C is an $(i,j)\mathcal{A}-$connected set, $C \subseteq B \subseteq \mathcal{A}cl(C)$ then C is an $(i,j)\mathcal{A}-$connected set.

20.3) Let (X, m_X^1, m_X^2) be a biminimal structure space.
If C is $(i,j)\mathcal{A}-$connected sets then $\mathcal{A}cl(C)$ is $(i,j)\mathcal{A}-$connected sets.

20.4) Let (X, m_X^1, m_X^2) be a biminimal structure space.
If C_α is $(i,j)\mathcal{A}-$connected for all $\alpha \in J$ and for $\beta, \gamma \in J$, $\beta \neq \gamma$, C_β and C_γ are not $(i,j)\mathcal{A}-$separated then $\bigcup\limits_{\alpha \in J} C_\alpha$ is $(i,j)\mathcal{A}-$connected as well.

20.5) Let (X, m_X^1, m_X^2) be a biminimal structure space and $C = \bigcup\limits_{\alpha \in J} C_\alpha$. If C_α is $(i,j)\mathcal{A}-$connected for all $\alpha \in J$ and $C_\beta \cap C_\gamma \neq \emptyset$ for all $\beta, \gamma \in J$ then C is $(i,j)\mathcal{A}-$connected.

20.6) Let (X, m_X^1, m_X^2) be a biminimal structure spaces and $C = \bigcup\limits_{\alpha \in J} C_\alpha$. If C_α is an $(i,j)\mathcal{A}-$connected for all $\alpha \in J$ and $\bigcap\limits_{\alpha \in J} C_\alpha \neq \emptyset$ then C is an $(i,j)\mathcal{A}-$connected.

21) Let (X, m_X^1, m_X^2) be a biminimal structure space and (X, m_X^1, m_X^2) is a $T_{\mathcal{A}}-$space, then the following statements are equivalent:

(1) The space X is $(i,j)\mathcal{A}-$connected sets;

(2) If $X = G_1 \cup G_2$, $G_1 \cap G_2 = \emptyset$, G_1 and G_2 are $(i,j)m_X - \mathcal{A}-$set then either $G_1 = \emptyset$ or $G_2 = \emptyset$;

(3) If $X = F_1 \cup F_2$, $F_1 \cap F_2 = \emptyset$, F_1 and F_2 are $(i,j)m_X - \mathcal{A}^C-$set then either $F_1 = \emptyset$ or $F_2 = \emptyset$;

(4) If $H \subseteq X$ is both $(i,j)m_X - \mathcal{A}-$set and $(i,j)m_X - \mathcal{A}^C-$set then either $H = \emptyset$ or $H = X$.

22) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. Let $f : X \to Y$, we will say that f is $(i,j)(\mathcal{A}_X, \mathcal{A}_Y)-$continuous iff $f^{-1}(W) \in (i,j)-\mathcal{A}(X, m_X^1, m_X^2)$ for all $W \in (i,j) - \mathcal{A}(Y, m_Y^1, m_Y^2)$.

From the above definitions, I have the following theorems are derived:

22.1) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be $T_{\mathcal{A}}-$ spaces.
If f is $(i,j)(\mathcal{A}_X, \mathcal{A}_Y)-$continuous and $V, W \subseteq Y$ are $(i,j)\mathcal{A}-$separated then $f^{-1}(V)$ and $f^{-1}(W)$ are $(i,j)\mathcal{A}-$separated.

22.2) Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be $T_{\mathcal{A}}-$spaces. If $C \subseteq X$ is $(i,j)\mathcal{A}-$connected and f is $(i,j)(\mathcal{A}_X, \mathcal{A}_Y)-$ continuous then $f(C)$ is $(i,j)\mathcal{A}-$connected.

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