PROPERTIES OF (m, μ)-PRECONTINUOUS FUNCTIONS

BY

KITTIYA WANWONGSA

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mathematics Education at Mahasarakham University

August 2014

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ABSTRACT

In this research, (m,μ)-precontinuous functions, almost (m,μ)- precontinuous functions and weakly (m,μ)- precontinuous functions, as functions from an m-space into a generalized topological space, are introduced. Some properties, characterizations and relationships between (m,μ)-precontinuous functions, almost (m,μ)-precontinuous functions and weakly (m,μ)-precontinuous functions are obtained.

Keywords: m-space, generalized topological space, (m,μ)- precontinuous function, almost (m,μ)- precontinuous function, weakly (m,μ)- precontinuous function.

ชื่อเรื่อง สมบัติของฟังก์ชันก่อนต่อเนื่อง (m,μ)
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บทคัดย่อ

งานวิจัยนี้ ได้นิยามฟังก์ชันก่อนต่อเนื่อง (m,μ), ฟังก์ชันเกือบก่อนต่อเนื่อง (m,μ) และฟังก์ชันก่อนต่อเนื่องอย่างอ่อน (m,μ) เป็นฟังก์ชันจากปริภูมิโครงสร้างเล็กสุดไปยังปริภูมิเชิงทอทอโลยีวางนัยทั่วไป จากการศึกษาได้สมบัติบางอย่าง ลักษณะเฉพาะ และความสัมพันธ์ระหว่างฟังก์ชันก่อนต่อเนื่อง (m,μ) ฟังก์ชันเกือบก่อนต่อเนื่อง (m,μ) และฟังก์ชันก่อนต่อเนื่องอย่างอ่อน (m,μ)

คำสำคัญ: ปริภูมิโครงสร้างเล็กสุด ปริภูมิเชิงทอพอโลยีวางนัยทั่วไป ฟังก์ชันก่อนต่อเนื่อง (m,μ) ฟังก์ชันเกือบก่อนต่อเนื่อง (m,μ) ฟังก์ชันก่อนต่อเนื่องอย่างอ่อน (m,μ)

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CHAPTER 1

INTRODUCTION

1.1 Background

The concepts of generalized neighborhood systems and generalized topological spaces were introduced by Á. Császár [1] in 2000. He also introduced the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces.

In 2000, V. Popa and T.Noiri [7] introduced and studied minimal structure. The notions of m_X -open set and m_X -closed set and characterized those sets using m_X -closure and m_X -interior operetors, respectively. The concept of m_X -preopen sets was introduced by E. Rosas[4] in 2009. In the same year, W. K. Min and Y. K. Kim [9] introduced and studied m -precontinuous functions from an m -spaces into a topological space. In 2010, C. Boonpok [2], [3] introduced (μ, m) -continuous functions, almost (μ, m) -continuous functions and weakly (μ, m) -continuous functions as functions from a generalized topological space into an m -space and investigated some their characterizations. Later, S. Pholdee, C. Boonpok and C. Viriyapong [7] introduced and studied (m, μ) -continuous functions, almost (m, μ) -continuous functions, weakly (m, μ) -continuous functions as functions from an m -space into a generalized topological space.

This thesis is divided into five chapter. The first chapter comprises an introduction which contains some historical remark concerning the research specialization. We also explain our motivation and outline the goals of the thesis here. In the second chapter, we give some definitions, notations and some known theorems that will be used in the later chapters. In the third and the fourth chapter, we give some definitions, notations and some interesting proposition of (m, μ) -precontinuous functions, almost (m, μ) -precontinuous functions and weakly (m, μ) -precontinuous functions, respective as functions from an m -space into a generalized topological space which is the fundamental properties for the last chapter and we give their characterization. In the last chapter, we draw conclusions based on the obtained results and outline a possible direction for further research.

1.2 Objectives of the research

1.2.1 To create and properties of (m,μ)-precontinuous function.

1.2.2 To create and properties of almost (m,μ)-precontinuous function.

1.2.3 To create and properties of weakly (m,μ)-precontinuous function.

1.2.4 The relationships between (m,μ)-precontinuous function, almost (m,μ)-precontinuous function, weakly (m,μ)-precontinuous function.

1.3 Research methodology

The research procedure of this thesis consists of the following step:

1.3.1 Criticism and possible extension of the literature review.

1.3.2 Doing research to investigate the main results.

1.3.3 Applying the results from 1.3.1 and 1.3.2 to the main results.

1.3.4 Making the conclusions and recommendations.

1.4 Scope of the Research

The scope of the study are:

1.4.1 Constructing (m,μ)-precontinuous function, alomst (m,μ)-precontinuous function and weakly (m,μ)-precontinuous function from an m-space into a generalized topological space.

1.4.2 Studying properties of (m,μ)-precontinuous function, alomst (m,μ)-precontinuous function and weakly (m,μ)-precontinuous function from an m-space into a generalized topological space and studying certain relationships between the above three functions.

1.4.3 Making the conclusions and recommendations.

CHAPTER 2

PRELIMINARIES

This chapter comprises the fundamental properties needed in the proof of each theorem in the study.

2.1 Generalized Topological Spaces

Definition 2.1.1 [1] Let X be a nonempty set and μ be a collection of subsets of X. Then μ is called a generalized topology (briefly GT) on X iff $\phi \in \mu$ and $G_i \in \mu$ for $i \in I \neq \phi$ implies $G = \bigcup_{i \in I} G_i \in \mu.$ We call the pair (X, μ) a generalized topological space (briefly GTS). The elements of μ are called μ-open sets and the complements are called μ-closed sets.

Example 2.1.2 Let $X = \{a, b, c\}$.

(1) Let $\mu_1 = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. Then μ_1 is a generalized topology on X. Thus $\phi, \{a\}, \{b\}$ and $\{a, b\}$ are μ-open sets, $X, \{b, c\}, \{a, c\}$ and $\{c\}$ are μ-closed sets.

(2) Let $\mu_2 = \{\phi, \{a\}, \{b\}\}$. Then μ_2 is not a generalized topology on X since $\{a\} \in \mu_2$ and $\{b\} \in \mu_2$ but $\{a\} \cup \{b\} = \{a, b\} \notin \mu_2$.

Definition 2.1.3 [1] Let X be a nonempty set and μ a generalized topology on X. For a subset A of X, the μ-closure of A, denoted by $c_\mu(A)$, and the μ-interior of A, denoted by $i_\mu(A)$, are defined as follows:

(1) $c_\mu(A) = \cap\{F \mid A \subseteq F, X - F \in \mu\}$;

(2) $i_\mu(A) = \cup\{G \mid G \subseteq A, G \in \mu\}$.

Theorem 2.1.4 [1] Let (X, μ) be a generalized topology space and $A \subseteq X$. Then

(1) $c_\mu(A) = X - i_\mu(X - A)$;

(2) $i_\mu(A) = X - c_\mu(X - A)$.

Proposition 2.1.5 [1] Let (X, μ) be a generalized topological space and $A \subseteq X$ Then

(1) $x \in i_\mu(A)$ if and only if there exists $V \in \mu$ such that $x \in V \subseteq A$;

(2) $x \in c_\mu(A)$ if and only if $V \cap A \neq \phi$ for every μ-open set V containing x.

Proposition 2.1.6 [1] Let (Y, μ) be a generalized topological space. For A and B of Y, the following properties hold:

(1) $c_\mu(Y - A) = Y - i_\mu(A)$ and $i_\mu(Y - A) = Y - c_\mu(A)$;

(2) if $Y - A \in \mu$, then $c_\mu(A) = A$ and if $A \in \mu$, then $i_\mu(A) = A$;

(3) if $A \subseteq B$, then $c_\mu(A) \subseteq c_\mu(B)$ and $i_\mu(A) \subseteq i_\mu(B)$;

(4) $A \subseteq c_\mu(A)$ and $i_\mu(A) \subseteq A$;

(5) $c_\mu(c_\mu(A)) = c_\mu(A)$ and $i_\mu(i_\mu(A)) = i_\mu(A)$.

Definition 2.1.7 [3] Let (X, μ) be a generalized topology space and $A \subseteq X$. Then A is said to be

(1) μ-semi-open if $A \subseteq c_\mu(i_\mu(A))$,

(2) μ-preopen if $A \subseteq i_\mu(c_\mu(A))$,

(3) μ-α-open if $A \subseteq i_\mu(c_\mu(i_\mu(A)))$,

(4) μ-β-open if $A \subseteq c_\mu(i_\mu(c_\mu(A)))$.

The complement of a μ-semi-open (resp., μ-preopen, μ-α-open, μ-β-open) set is called μ-semi-closed (resp., μ-preclosed, μ-α-closed, μ-β-closed).

Definition 2.3.8 [6] A subset A of (X, μ) is said to be

(i) μ-regular closed if $A = c_\mu(i_\mu(A))$

(ii) μ-regular open if $X - A$ is μ-regular closed

2.2 m-spaces

Definition 2.2.1 [8] Let X be a nonempty set and $P(X)$ the power set of X. A subfamily m_X of $P(X)$ is called a minimal structure (briefly m-structure) on X if $\phi \in m_X$ and $X \in m_X$.

By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and it is called an m-space. Each member of m_X is said to be m_X-open and the complement of an m_X-open set is said to be m_X-closed.

Definition 2.2.2 [8] Let X be a nonempty set and m_X a minimal structure on X. For a subset A of X, the m_X-closure of A, denoted by $m_X\text{-}Cl(A)$, and the m_X-interior of A, denoted by $m_X\text{-}Int(A)$ are defined as follows:

(1) $m_X\text{-}Cl(A) = \cap\{F : A \subseteq F, X - F \in m_X\}$;

(2) $m_X\text{-}Int(A) = \cup\{U : U \subseteq A, U \in m_X\}$.

Lemma 2.2.3 [8] Let X be a nonempty set and m an m_X-structure on X. For subset A and B of X, the following properties hold.

(1) $m_X\text{-}Cl(X - A) = X - m_X\text{-}Int(A)$ and $m_X\text{-}Int(X - A) = X - m_X\text{-}Cl(A)$;

(2) if $X - A \in m_X$, then $m_X\text{-}Cl(A) = A$ and if $A \in m_X$, then $m_X\text{-}Int(A) = A$;

(3) $m_X\text{-}Cl(\phi) = \phi$, $m_X\text{-}Cl(X) = X$, $m_X\text{-}Int(\phi) = \phi$ and $m_X\text{-}Int(X) = X$;

(4) if $A \subseteq B$, then $m_X\text{-}Cl(A) \subseteq m_X\text{-}Cl(B)$ and $m_X\text{-}Int(A) \subseteq m_X\text{-}Int(B)$;

(5) $A \subseteq m_X\text{-}Cl(A)$ and $m_X\text{-}Int(A) \subseteq A$;

(6) $m_X\text{-}Cl(m_X\text{-}Cl(A)) = m_X\text{-}Cl(A)$ and $m_X\text{-}Int(m_X\text{-}Int(A)) = m_X\text{-}Int(A)$.

Lemma 2.2.4 [8] Let X be a nonempty set with a minimal structure m_X and A a subset of X. Then $x \in m\text{-}Cl(A)$ if and only if $U \cap A \neq \phi$ for every m_X-open set U containing x.

Definition 2.2.5 [3] An m-structure m_X on a nonempty set X is said to have property B if the union of any family of subsets belong to m_X belong to m_X.

Lemma 2.2.6 [5] Let X be a nonempty set and m_X an m-structure on X satisfying property B. For a subsets A of X, the following properties hold.

(1) $A \in m_X$ if and only if $m_X\text{-}Int(A) = A$;

(2) A is m_X-closed if and only if $m_X\text{-}Cl(A) = A$;

(3) $m_X\text{-}Int(A) \in m_X$ and $m_X\text{-}Cl(A)$ is m_X-closed.

Definition 2.2.7 [2] A subset A of a m-space (X, m_X) is said to be

(1) m_X-regular open if $A = m_X\text{-}Int(m_X\text{-}Cl(A))$;

(2) m_X-semi-open if $A \subseteq m_X\text{-}Cl(m_X\text{-}Int(A))$;

(3) m_X-preopen if $A \subseteq m_X\text{-}Int(m_X\text{-}Cl(A))$;

(4) $m_X\text{-}\alpha$-open if $A \subseteq m_X\text{-}Int(m_X\text{-}Cl(m_X\text{-}Int(A)))$;

(5) $m_X\text{-}\beta$-open if $A \subseteq m_X\text{-}Cl(m_X\text{-}Int(m_X\text{-}Cl(A)))$.

2.3 (m,μ)-continuous functions.

Definition 2.3.1 [7] A function $f:(X, m_X) \to (Y, \mu)$ is said to be (m,μ)-continuous at a point $x \in X$ if for each μ-open set V containing $f(x)$, there exists an m_X-open set U containing x such that $f(U) \subseteq V$. A function $f:(X, m_X) \to (Y, \mu)$ is said to be (m,μ)-continuous if it has this property at each point $x \in X$.

Theorem 2.3.2 [7] For a function $f:(X, m_X) \to (Y, \mu)$, the following properties are equivalent:

(1) f is (m,μ)-continuous at $x \in X$;

(2) $x \in m_X\text{-}Int(f^{-1}(V))$ for every $V \in \mu$ containing $f(x)$;

(3) $x \in f^{-1}(c_\mu(f(A)))$ for every subset A of X with $x \in m_X\text{-}Cl(A)$;

(4) $x \in f^{-1}(c_\mu(B))$ for every subset B of Y with $x \in m_X\text{-}Cl(f^{-1}(B))$;

(5) $x \in m_X\text{-}Int(f^{-1}(B))$ for every subset B of Y with $x \in f^{-1}(i_\mu(B))$;

(6) $x \in f^{-1}(F)$ for every μ-closed set F of Y such that $x \in m_X\text{-}Cl(f^{-1}(F))$.

Theorem 2.3.3 [7] For a function $f:(X,m_X)\to(Y,\mu),$ the following properties are equivalent:

(1) f is (m,μ)-continuous;

(2) $f^{-1}(V)$ is m_X-open in X for every μ-open set V of Y;

(3) $f(m_X\text{-}Cl(A))\subseteq c_\mu(f(A))$ for every subset A of X;

(4) $m_X\text{-}Cl(f^{-1}(B))\subseteq f^{-1}(c_\mu(B))$ for every subset B of Y;

(5) $f^{-1}(i_\mu(B))\subseteq m_X\text{-}Int(f^{-1}(B))$ for every subset B of Y;

(6) $f^{-1}(F)$ is m_X-closed in X for every μ-closed set F of Y.

Definition 2.3.4 [7] A function $f:(X,m_X)\to(Y,\mu)$ is said to be almost (m,μ)-continuous at a point $x\in X$ if for each μ-open set V containing $f(x),$ there exists an m_X-open set U containing x such that $f(U)\subseteq i_\mu(c_\mu(V)).$ A function $f:(X,m_X)\to(Y,\mu)$ is said to be almost (m,μ)-continuous if it has this property at each point $x\in X$.

Theorem 2.3.5 [7] For a function $f:(X,m)\to(Y,\mu),$ the following properties are equivalent:

(1) f is almost (m,μ)-continuous;

(2) $f^{-1}(V)\subseteq m_X\text{-}Int(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open set V of Y;

(3) $m_X\text{-}Cl(f^{-1}(c_\mu(i_\mu(F))))\subseteq f^{-1}(F)$ for every μ-closed subset F of Y;

(4) $m_X\text{-}Cl(f^{-1}(c_\mu(i_\mu(c_\mu(B)))))\subseteq f^{-1}(c_\mu(B))$ for every subset B of Y;

(5) $f^{-1}(i_\mu(B))\subseteq m_X\text{-}Int(f^{-1}(i_\mu(c_\mu(i_\mu(B)))))$ for every subset B of Y;

(6) $f^{-1}(V)$ is m_X-open in X for every μ-regular open subset V of Y;

(7) $f^{-1}(F)$ is m_X-closed in X for every μ-regular closed subset V of Y.

Theorem 2.3.6 [7] For a function $f:(X,m_X)\to(Y,\mu),$ the following properties are equivalent:

(1) f is almost (m,μ)-continuous;

(2) $m_X\text{-}Cl(f^{-1}(U))\subseteq f^{-1}(c_\mu(U))$ for every μ-open subset U of Y;

(3) $m_X\text{-}Cl(f^{-1}(U)) \subseteq f^{-1}(c_\mu(U))$ for every μ-semiopen subset U of Y;

(4) $f^{-1}(U) \subseteq m_X\text{-}Int(f^{-1}(i_\mu(c_\mu(U))))$ for every μ-preopen subset U of Y.

Definition 2.3.7 [7] A function $f:(X,m_X) \to (Y,\mu)$ is said to be weakly (m,μ)-continuous at a point $x \in X$ if for each μ-open set V containing $f(x)$, there exists an m_X-open set U containing x such that $f(U) \subseteq c_\mu(V)$. A function $f:(X,m) \to (Y,\mu)$ is said to be weakly (m,μ)-continuous if it has this property at each point $x \in X$.

Theorem 2.3.8 [7] A function $f:(X,m_X) \to (Y,\mu)$ is weakly (m,μ)-continuous at x if and only if for each μ-open set V containing $f(x)$, $x \in m_X\text{-}Int(f^{-1}(c_\mu(V)))$.

Theorem 2.3.9 [6] A function $f:(X,m_X) \to (Y,\mu)$ is weakly (m,μ)-continuous if and only if $f^{-1}(V) \subseteq m_X\text{-}Int(f^{-1}(c_\mu(V)))$ for every μ-open set V of Y.

Theorem 2.3.10 [7] For a function $f:(X,m_X) \to (Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-continuous;

(2) $f^{-1}(V) \subseteq m_X\text{-}Int(f^{-1}(c_\mu(V)))$ for every μ-open subset V of Y;

(3) $m_X\text{-}Cl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(F)$ for every μ-closed subset F of Y;

(4) $m_X\text{-}Cl(f^{-1}(i_\mu(c_\mu(A)))) \subseteq f^{-1}(c_\mu(A))$ for every subset A of Y;

(5) $f^{-1}(i_\mu(A)) \subseteq m_X\text{-}Int(f^{-1}(c_\mu(i_\mu(A))))$ for every subset A of Y;

(6) $m_X\text{-}Cl(f^{-1}(V)) \subseteq f^{-1}(c_\mu(V))$ for every μ-open subset V of Y.

Theorem 2.3.11 [7] For a function $f:(X,m_X) \to (Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-continuous;

(2) $m_X\text{-}Cl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(F)$ for every μ-regular closed subset F of Y;

(3) $m_X\text{-}Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-open subset G of Y;

(4) $m_X\text{-}Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-semiopen subset G of Y.

Theorem 2.3.12 [7] For a function $f:(X,m)\to(Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-continuous;

(2) $m_X\text{-}Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-preopen subset G of Y;

(3) $m_X\text{-}Cl(f^{-1}(G)) \subseteq f^{-1}(c_\mu(G))$ for every μ-preopen subset G of Y;

(4) $f^{-1}(G) \subseteq m_X\text{-}Int(f^{-1}(c_\mu(G)))$ for every μ-preopen subset G of Y.

CHAPTER 3

Properties of (m,μ)-precontinuous functions

3.1 Properties of m_X -preclosure and m_X -preinterior

Definition 3.1.1 Let X be a nonempty set with m_x an m-structure on X. For a subset A of X, the m_X -pre-closure of A, denote by m_X - $pCl(A)$ and the m_X -preinterior of A, denote by m_X - $pInt\,(A)$, are defined as follow.

(1) m_X - $pCl(A) = \cap\{F: A \subseteq F, X - F$ is m_X -preclosed $\}$;

(2) m_X - $pInt\,(A) = \cup\ \{G: G \subseteq A, G$ is m_X -preopen $\}$;

Example 3.1.2 Let $X = \{1,2,3\}$ $m_X = \{\phi, \{1\}, \{2\}, \{1,2\}, X\}$ and $A = \{2\}$.Then m_X - $pCl(A) = \{2,3\}$ and m_X - $pInt\,(A) = \{2\}$.

Theorem 3.1.3 Let (X, m_X) be an m -space and $A, B \subseteq X$. If $A \subseteq B$, then m_X - $pCl \subseteq m_X$ - $pCl(B)$, m_X - $pInt(A) \subseteq m_X$ - $pInt(B)$.

proof Assume that $A \subseteq B$. Then

$\mathscr{A} = \{F : F$ is m_X -preclosed and $A \subseteq F\} \supseteq \mathscr{B} = \{K : K$ is m_X -preclosed and $B \subseteq K\}$.

This m_X - $pCl(A) \subseteq m_X$ - $pCl(B)$. Next we will show that m_X - $Int(A) \subseteq m_X$ - $Int(B)$. Since $\mathscr{C} = \{G : G$ is m_X -preopen and $G \subseteq A\} \subseteq \mathscr{D} = \{V : V$ is m_X -preopen $V \subseteq B\}$,

m_X - $pInt(A) \subseteq m_X$ - $pInt(B)$.

Theorem 3.1.4 Let (X, m_X) be an m -space and let $\{A_\gamma : \gamma \in J\}$ be a family of subset of X. Then the following statements hold.

(1) If A_γ is m_X -preopen for all $\gamma \in J$, then $\bigcup_{\gamma \in J} A_\lambda$ is m_X -preopen.

(2) If A_γ is m_X -preclosed for all $\gamma \in J$, then $\bigcap_{\gamma \in J} A_\gamma$ is m_X -preclosed.

Proof (1) Assume that A_γ is m_X -preopen for all $\gamma \in J$. Let $\lambda \in J$.

Then $A_\lambda \subseteq \bigcup_{\gamma \in J} A_\gamma$, and so m_X - $Int(\ m_X$ - $Cl(A_\lambda)) \subseteq m_X$ - $Int(m_X$ - $Cl(\bigcup_{\gamma \in J} A_\gamma))$.

Since A_λ is m_X -preopen, $A_\lambda \subseteq m_X$ - $Int(m_X$ - $Cl(\bigcup_{\gamma \in J} A_\gamma))$. This implies

$\bigcup_{\gamma\in J} A_\gamma \subseteq m_X\text{-}Int(m_X\text{-}Cl(\bigcup_{\gamma\in J} A_\gamma))$. Then $\bigcup_{\gamma\in J} A_\gamma$ is m_X -preopen.

(2) Assume that A_γ is m_X -preclosed for all $\gamma \in J$. Then $X - A_\gamma$ is m_X -preopen for all $\gamma \in J$. By (1), $\bigcup_{\gamma\in J}(X - A_\gamma)$ is m_X -preopen. Since $\bigcup_{\gamma\in J}(X - A_\gamma) = X - (\bigcap_{\gamma\in J} A_\gamma)$, $X - (\bigcap_{\gamma\in J} A_\gamma)$ is m_X -preopen. Then $\bigcap_{\gamma\in J} A_\gamma$ is m_X -preclosed.

Lemma 3.1.5 Let X be a nonempty set with a minimal structure m_X on X and A subset of X. Then $x \in m_X\text{-}pCl(A)$ if and only if $A \cap U \neq \phi$, U is m_X -preopen containing x.

Proof ($\Rightarrow$) Suppose there exists an m_X -preopen set U containing x such that $U \cap A = \phi$. Then $X - U$ is m_X -preclosed and $A \subseteq X - U$.
Since $x \notin X - U$, $x \notin m_X\text{-}pCl(A)$.

($\Leftarrow$) Suppose that $x \notin m_X\text{-}pCl(A)$. Then $x \notin F$ for some m_X -preclosed set F with $A \subseteq F$. Thus $X - F$ is m_X -preopen and $x \in X - F$. Futhermore, $(X - F) \cap A \neq \phi$.

Lemma 3.1.6 Let X be nonempty set with an m -structure m_X on X.
For a subset A of X, the following properties hold:

(1) $X - m_X\text{-}pInt(A) = m_X\text{-}pCl(X - A)$;

(2) $X - m_X\text{-}pCl(A) = m_X\text{-}pInt(X - A)$.

Proof (1) ($\subseteq$) Assume that $x \notin m_X\text{-}pCl(X - A)$. Then there exists an m_X -preclosed set F such that $x \notin F$ and $X - A \subseteq F$. Thus $X - F$ is an m_X -preopen set containing x such that $X - F \subseteq A$. Hence $x \in m_X\text{-}pInt(A)$, and so $x \notin X - m_X\text{-}pInt(A)$.

($\supseteq$) Assume that $x \notin X - m_X\text{-}pInt(A)$. Then $x \in m_X\text{-}pInt(A)$. Thus $x \in G$ for some m_X -preopen set G with $G \subseteq A$. Hence $X - G$ is m_X -preclosed such that $x \notin X - G$ and $X - A \subseteq X - G$. then $x \notin m_X\text{-}pCl(X - A)$.

(2) Since $X - A \subseteq X$, by (1), we obtain that

$X - m_X$ - $pInt(X - A) = m_X$ - $pCl(X - (X - A)) = m_X$ - $pCl(A)$.

Then $X - m_X$ - $pCl(A) = X - (X - m_X$ - $pInt(X - A)) = m_X$ - $pInt(X - A)$.

Lemma 3.1.7 Let X be nonempty set with m_X an m-structure on X. For a subset A of X, the following properties hold:

(1) m_X - $pInt(A)$ is m_X-preopen;

(2) m_X - $pCl(A)$ is m_X-preclosed;

(3) A is m_X-preopen if and only if $A = m_X$ - $pInt(A)$;

(4) A is m_X-preclosed if and only if $A = m_X$ - $pCl(A)$;

(5) m_X - $pCl(m_X$ - $pCl(A) = m_X$ - $pCl(A)$;

(6) m_X - $pInt(m_X$ - $Int(A) = m_X$ - $pInt(A)$.

Proof (1) follows from Theorem 3.1.2 (1).

(2) follows from Theorem 3.1.6 (2).

(3)($\Rightarrow$) Assume A is m_X-preopen. Then $A \in \{G : G$ is m_X-preopen $G \subseteq A\}$. Thus $A \subseteq \cup\{G : G$ is m_X-preopen $G \subseteq A\} = m_X$ - $pInt(A)$. Since m_X - $pInt(A) \subseteq A$, $A = m_X$ - $pInt(A)$.

($\Leftarrow$) Assume that $A = m_X$ - $pInt(A)$. By (1), A is m_X-preopen.

(4)($\Rightarrow$) Assume A is m_X-preclosed.

Then $A \in \{F : F$ is m_X-preclosed $A \subseteq F\}$. Thus $A \supseteq \cap\{F : F$ is m_X-preclosed $A \subseteq F\} = m_X$ - $pCl(A)$. Since $A \subseteq m_X$ - $pCl(A)$, $A = m_X$ - $pCl(A)$.

($\Leftarrow$) Assume $A = m_X$ - $pCl(A)$. By (2), A is m_X-preclosed.

(5) By (2) and (4), m_X - $Cl(m_X$ - $Cl(A)) = m_X$ - $Cl(A)$.

(6) By (1) and (3), m_X - $Int(m_X$ - $Int(A) = m_X$ - $Int(A)$.

3.2 (m,μ)-precontinuous functions

Definition 3.2.1 Let (X,m_X) be an m -space and ($Y,\ \mu$) be a generalized topological space. A function $f:(X,m_X)\rightarrow(Y,\mu)$ is said to be (m,μ)- precontinuous at a point $x\in X$ if for each μ -open set V containing $f(x)$, there exists an m_X -preopen set U containing x such that $f(U)\subseteq V$. A function $f:(X,m_X)\rightarrow(Y,\mu)$ is said to be (m,μ)-precontinuous if it has this property at each point $x\in X$.

Example 3.2.2 Let $X=\{1,2,3\}$, $m_X=\{\phi,\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},X\}$ and $Y=\{a,b,c\}$, $\mu_Y=\{\phi,\{a\},\{b\},\{a,b\}\}$. Define $f:(X,m_X)\rightarrow(Y,\mu)$ as follows : $f(1)=a, f(2)=b$ and $f(3)=a$ Then f is (m,μ)-precontinuous.

Theorem 3.2.3 For a function $f:(X,m_X)\rightarrow(Y,\mu)$ and $x\in X$, the following properties are equivalent:

(1) f is (m,μ)-precontinuous at $x\in X$;

(2) $x\in m_X$ - $pInt(f^{-1}(V))$ for every $V\in\mu$ containing $f(x)$;

(3) $x\in f^{-1}(c_\mu(f(A)))$ for every subset A of X with $x\in m_X$ - $pCl(A)$;

(4) $x\in f^{-1}(c_\mu(B))$ for every subset B of Y with $x\in m_X$ - $pCl(f^{-1}(B))$;

(5) $x\in m_X$ - $pInt(f^{-1}(B))$ for every subset B of Y with $x\in f^{-1}(i_\mu(B))$;

(6) $x\in f^{-1}(F)$ for every μ -closed set F of Y such that $x\in m_X$ - $pCl(f^{-1}(F))$.

Proof (1) $\Rightarrow$(2) : Let V be any μ -open subset of Y containing $f(x)$. By assumption, there exists an m_X -preopen subset U of X containing x such that $f(U)\subseteq V$, and so $x\in U\subseteq f^{-1}(V)$. Since U is an m_X -preopen set, we have $x\in m_X$ - $pInt(f^{-1}(V))$.

(2) $\Rightarrow$(1): Let V be any μ -open subset of Y containing $f(x)$. By (2), $x\in m_X$ - $pInt(f^{-1}(V))$ and hence there exists an m_X -preopen set U containing x such that $x\in U\subseteq f^{-1}(V)$. Therefore, $f(U)\subseteq f(f^{-1}(V))\subseteq V$, and so f is (m,μ)-precontinuous at x.

(2) $\Rightarrow$(3): Let A be a subset of X such that $x \in m_X$ - $pCl(A)$ and let V be any μ-open subset of Y containing $f(x)$. By (2), we have $x \in m_X$ - $pInt(f^{-1}(V))$. Then, there exists an m_X-preopen subset U of X containing x such that $x \in U \subseteq f^{-1}(V)$. Since $x \in m_X$ - $pCl(A)$, $U \cap A \neq \phi$. Thus $\phi \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. Since V is μ-open containing $f(x)$, $f(x) \in c_\mu(f(A))$, and hence $x \in f^{-1}(c_\mu(f(A)))$.

(3) $\Rightarrow$(4): Let B be any subset of Y such that $x \in m_X$ - $pCl(f^{-1}(B))$. By (3), $x \in f^{-1}(c_\mu(f(f^{-1}(B))) \subseteq f^{-1}(c_\mu(B))$. Hence, $x \in f^{-1}(c_\mu(B))$.

(4) $\Rightarrow$(5): Let B be any subset of Y such that $x \notin m_X$ - $pInt(f^{-1}(B))$. Then $x \in X - (m_X$ - $pInt(f^{-1}(B)) = m_X$ - $pCl(X - f^{-1}(B)) = m_X$ - $pCl(f^{-1}(Y - B))$. By(4), we have $x \in f^{-1}(c_\mu(Y - B)) = f^{-1}(Y - i_\mu(B)) = X - f^{-1}(i_\mu(B))$. Hence, $x \notin f^{-1}(i_\mu(B))$.

(5) $\Rightarrow$(6): Let F be a μ-closed subset of Y such that $x \notin f^{-1}(F)$. Then $x \in X - f^{-1}(F) = f^{-1}(Y - F) = f^{-1}(i_\mu(Y - F))$ because $Y - F$ is μ-open. By (5), $x \in m_X$ - $pInt(f^{-1}(Y - F)) = m_X$ - $pInt(X - f^{-1}(F)) = X - m_X$ - $pCl(f^{-1}(F))$. Hence, $x \notin m_X$ - $pCl(f^{-1}(F))$.

(6) $\Rightarrow$(2): Let V be any μ-open subset of Y containing $f(x)$. Suppose that $x \notin m_X$ - $pInt(f^{-1}(V))$. Then $x \in X - m_X$ - $pInt(f^{-1}(V)) = m_X$ - $pCl(X - f^{-1}(V)) = m_X$ - $pCl(f^{-1}(Y - V))$. By (6), $x \in f^{-1}(Y - V) = X - f^{-1}(V)$. Hence $x \notin f^{-1}(V)$. This is a contradiction

Theorem 3.2.4 For a function $f : (X, m_X) \to (Y, \mu)$, the following properties are equivalent:

(1) f is (m, μ)-precontinuous;

(2) $f^{-1}(V)$ is m_X-preopen in X for every μ-open set V of Y;

(3) $f(m_X$ - $pCl(A)) \subseteq c_\mu(f(A))$ for every subset A of X;

(4) m_X - $pCl(f^{-1}(B)) \subseteq f^{-1}(c_\mu(B))$ for every subset B of Y;

(5) $f^{-1}(i_\mu(B)) \subseteq m_X$ - $pInt(f^{-1}(B))$ for every subset B of Y;

(6) $f^{-1}(F)$ is m_X -preclosed in X for every μ -closed set F of Y.

Proof (1) $\Rightarrow$(2): Let $V \in \mu$ and let $x \in f^{-1}(V)$. Since f is (m,μ)-precontinuous, there exists an m_X -preopen subset U of X containing x such that $f(U) \subseteq V$. Since U is m_X -preopen, we have $x \in m_X$ - $pInt(f^{-1}(V))$. Hence $f^{-1}(V) = m_X$ - $pInt(f(V))$, and so $f^{-1}(V)$ is m_X -preopen in X.

(2) $\Rightarrow$(1): Let $x \in X$ and V be a μ -open subset of Y containing f(x). Then $x \in f^{-1}(V)$. By (2), $x \in m_X$ - $pInt(f^{-1}(V))$ and hence there exists an m_X -preopen subset U of X containing x such that $x \in U \subseteq f^{-1}(V)$. Therefore $f(U) \subseteq V$ and so f is (m,μ)-precontinuous at x.

(2) $\Rightarrow$(3): Let A be any subset of X. Let $x \in m_X$ - $pCl(A)$ and $V \in \mu$ containing f(x). By (2), we get that $x \in m_X$ - $pInt(f^{-1}(V))$. Thus there exists an m_X -preopen subset U of X such that $x \in U \subseteq f^{-1}(V)$. Since $x \in m_X$ - $pCl(A)$, $U \cap A \neq \phi$. Then $\phi \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. Since V is μ - open containing $f(x)$, $f(x) \in c_\mu(f(A))$, and so $x \in f^{-1}(c_\mu(f(A))$. Hence, m_X - $pCl(A) \subseteq f^{-1}(c_\mu(f(A)))$. Then $f(m_X$ - $pCl(A)) \subseteq c_\mu(f(A))$.

(3) $\Rightarrow$(4): Let B be any subset of Y. By (3), $f(m_X$ - $pCl(f^{-1}(B))) \subseteq c_\mu(f(f^{-1}(B)))$. Hence, m_X - $pCl(f^{-1}(B)) \subseteq f^{-1}(c_\mu(B))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y. By (4), we have

$$
\begin{aligned}
X - m_X\text{ - } pInt(f^{-1}(B)) &= m_X\text{ - } pCl(X - f^{-1}(B)) \\
&= m_X\text{ - } pCl(f^{-1}(Y - B)) \\
&\subseteq f^{-1}(c_\mu(Y - B)) \\
&= f^{-1}(Y - i_\mu(B)) \\
&= X - f^{-1}(i_\mu(B)).
\end{aligned}
$$

Hence, $f^{-1}(i_\mu(B)) \subseteq m_X\text{-}\ pInt(f^{-1}(B))$.

(5) $\Rightarrow$(6): Let F be any μ-closed subset of Y. Then

$Y - F = i_\mu(Y - F)$. By(5); $X - f^{-1}(F) = f^{-1}(Y - F)$

$$= f^{-1}(i_\mu(Y - F))$$

$$\subseteq m_X\text{-}\ pInt(f^{-1}(Y - F))$$

$$= m_X\text{-}\ pInt(X - f^{-1}(F))$$

$$= X - m_X\text{-}\ pCl(f^{-1}(F)).$$

Hence, $m_X\text{-}\ pCl(f^{-1}(F)) \subseteq f^{-1}(F)$.

(6) $\Rightarrow$ (2): It is obvious.

3.3 Almost (m,μ)-precontinuous function

In this section, we introduce a new type of continuity called almost (m,μ)-precontinuous which is weaker than (m,μ)-precontinuous.

Definition 3.3.1 Let (X, m_X) be an m-space and (Y,μ) a generalized topological spaces. A function $f:(X,m_X)\to(Y,\mu)$ is said to be almost (m,μ)-precontinuous at a point $x \in X$ if for each μ-open set V containing $f(x)$, there exists an m_X-preopen U containing x such that $f(U) \subseteq i_\mu(c_\mu(V))$. A function $f:(X,m_X)\to(Y,\mu)$ is said to be almost (m,μ)-precontinuous if property at each point $x \in X$.

Remark 3.3.2 From the above definitions, we have a following implication but the reverse relation may not be true in general:

(m,μ)-precontinuous $\Rightarrow$ almost (m,μ) precontinuous.

Example 3.3.3 Let $X=\{1,2\}, m_X=\{\phi,\{1\},X\}$ and $Y=\{a,b\}, \mu=\{\phi,\{a\},Y\}$. Define $f:(X,m_X)\to(Y,\mu)$ as follows : $f(1)=a$ and $f(2)=b$. Then, f is almost (m,μ)-precontinuous but it is not (m,μ)-precontinuous.

Theorem 3.3.4 For a function $f:(X,m_X)\to(Y,\mu)$ and $x\in X$, the following properties are equivalent:

(1) f is almost (m,μ)-precontinuous at $x\in X$;

(2) $x\in m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open set V containing $f(x)$;

(3) $x\in m_X$ - $pInt(f^{-1}(V))$ for every μ-regular open set V containing $f(x)$;

(4)For every μ-regular open set V containing $f(x)$, there exists an m_X-preopen set U containing x such that $f(U)\subseteq V$.

Proof (1) $\Rightarrow$(2): Let V be any μ-open subset of Y containing $f(x)$. Then there exists an m_X-preopen subset U of X containing x such that $f(U)\subseteq c_\mu(i_\mu(V))$. Thus $x\in U\subseteq f^{-1}(i_\mu(c_\mu(V)))$, and so $x\in m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(V))))$.

(2) $\Rightarrow$(3): Let V be any μ-regular open subset of Y containing $f(x)$. Since $V=i_\mu(c_\mu(V))$ and by (2), we have $x\in m_X$ - $pInt(f^{-1}(V))$.

(3) $\Rightarrow$(4): Let V be any μ-regular open subset of Y containing $f(x)$. By (3), $x\in m_X$ - $pInt(f^{-1}(V))$. Then there exists an m_X-preopen set U containing x such that $U\subseteq f^{-1}(V)$.

(4) $\Rightarrow$(1): Let V be any μ-open subset of Y containing $f(x)$. Then $f(x)\in V\subseteq i_\mu(c_\mu(V))$. Since $i_\mu(c_\mu(V))$ is μ-regular open, by (4), there exists an m_X-preopen set U containing x such that $f(U)\subseteq i_\mu(c_\mu(V))$. Hence, f is almost (m,μ)-precontinuous at x.

Theorem 3.3.5 For a function $f:(X,m_X)\to(Y,\mu)$ the following properties are equivalent:

(1) f is almost (m,μ)-precontinuous;

(2) $f^{-1}(V)\subseteq m_X\text{-}pInt(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open set V of Y;

(3) $m_X\text{-}pCl(f^{-1}(c_\mu(i_\mu(F)))\subseteq f^{-1}(F)$ for every μ-closed subset F of Y;

(4) $m_X\text{-}pCl(f^{-1}(c_\mu(i_\mu(c_\mu(B)))))\subseteq f^{-1}(c_\mu(B))$ for every subset B of Y;

(5) $f^{-1}(i_\mu(B))\subseteq m_X\text{-}pInt(f^{-1}(i_\mu(c_\mu(i_\mu(B)))))$ for every subset B of Y;

(6) $f^{-1}(V)$ is m_X-preopen in X for every μ-regular open subset V of Y;

(7) $f^{-1}(F)$ is m_X-preclosed in X for every μ-regular closed subset F of Y.

proof. (1) $\Rightarrow$(2): Let V be any μ-open subset of Y and $x\in f^{-1}(V)$. By (1), there exists an m_X-preopen subset U of X containing x such that $f(U)\subseteq i_\mu(c_\mu(V))$. Thus $x\in m_X\text{-}(f^{-1}(i_\mu(c_\mu(V)))$. Then $f^{-1}(V)\subseteq m_X\text{-}pInt(f^{-1}(i_\mu(c_\mu(V))))$.

(2)$\Rightarrow$(3): Let F be any μ-closed subset of Y. By (2), we have

$$\begin{aligned}
X- f^{-1}(F) &= f^{-1}(Y- F)\\
&\subseteq m_X\text{-}pInt(f^{-1}(c_\mu(i_\mu(Y- F))))\\
&= m_X\text{-}pInt(f^{-1}(Y-(i_\mu(c_\mu(F))))\\
&= m_X\text{-}pInt(X- f^{-1}(i_\mu(c_\mu(F))))\\
&= X- m_X\text{-}pCl(f^{-1}(i_\mu(c_\mu(F))).
\end{aligned}$$

This implies that $m_X\text{-}pCl(f^{-1}(i_\mu(c_\mu(F))))\subseteq f^{-1}(F)$.

(3)$\Rightarrow$(4): Let B be a subset of Y. Since $c_\mu(B)$ is μ-closed and by (3), we have $m_X\text{-}pCl(f^{-1}(c_\mu(i_\mu(c_\mu(B)))))\subseteq f^{-1}(c_\mu(B))$.

(4) $\Rightarrow$(5): Let B be any subset of Y. By (4), we obtain that $X- f^{-1}(c_\mu(Y\text{-}B)))\subseteq X-(m_X\text{-}pCl(f^{-1}(c_\mu(i_\mu(c_\mu(Y- B)))))$. Then $f^{-1}(i_\mu(B))\subseteq m_X\text{-}pInt(f^{-1}(i_\mu(c_\mu(i_\mu(B))))$.

(5) $\Rightarrow$(6): Let V be any μ-regular open subset of Y. Since $i_\mu(c_\mu(i_\mu(V))) = V$ and by (5), $f^{-1}(V) \subseteq m_X$ - $pInt(f^{-1}(V))$. Then, $f^{-1}(V) = m_X$ - $pInt(f^{-1}(V))$ is m_X-preopen.

(6) $\Rightarrow$(7): Let F be any μ-regular closed subset of Y. By (6), $X - f^{-1}(F) = f^{-1}(Y - F)$ is m_X-preopen in X. Then $f^{-1}(F)$ is m_X-preclosed.

(7) $\Rightarrow$(1): Let $x \in X$ and let V be any μ-regular open subset of Y containing $f(x)$. By (7), $X - f^{-1}(V) = f^{-1}(Y - V) = m_X$- $pCl(f^{-1}(Y - V))$ $= X - m_X$- $pInt(f^{-1}(V))$. Since $x \in f^{-1}(V) = m_X$- $pInt(f^{-1}(V))$, there exists an m_X-preopen set U containing x such that $U \subseteq f^{-1}(V)$. Hence, by Theorem 3.3.1(4), f is almost (m, μ)-precontinuos at x Then f is almost (m, μ)-precontinuous.

Theorem 3.3.6 For a function $f : (X, m_X) \to (Y, \mu)$, the following properties are equivalent:

(1) f is almost (m, μ)-precontinuous.

(2) m_X- $pCl(f^{-1}(U)) \subseteq f^{-1}(c_\mu(U))$ for every μ- β-open subset U of Y;

(3) m_X- $pCl(f^{-1}(U)) \subseteq f^{-1}(c_\mu(U))$ for every μ-semi-open subset U of Y;

(4) $f^{-1}(U) \subseteq m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(U))))$ for every μ-preopen subset U of Y.

Proof. (1) $\Rightarrow$(2): Let U be any μ- β-open subset of Y. Since $c_\mu(U)$ is μ-regular closed, by Theorem 3.2.2(7), m_X - $pCl(f^{-1}(c_\mu(U))) = f^{-1}(c_\mu(U))$. Thus, m_X- $pCl(f^{-1}(U)) \subseteq m_X$ - $pCl(f^{-1}(c_\mu(U))) = f^{-1}(c_\mu(U))$.

(2) $\Rightarrow$(3): It follows from the fact that every μ-semi-open set is μ- β-open.

(3) $\Rightarrow$(1): Let F be any μ-regular closed subset of Y. Since F is μ-semi-open and by (3), we have m_X - $pCl(f^{-1}(F)) \subseteq f^{-1}(c_\mu(F)) = f^{-1}(F)$. Thus, by Theorem 3.3.2(7), f is almost (m, μ)-precontinuous.

(1) $\Rightarrow$(4): Let U be any μ-preopen subset of Y , Then $U \subseteq i_\mu(c_\mu(U))$ and $i_\mu(c_\mu(U))$ is μ-regular open. By Theorem 3.3.2 (6), we have $f^{-1}(i_\mu(c_\mu(U))) = m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(U))))$. Thus, $f^{-1}(U) \subseteq m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(U))))$.

(4) $\Rightarrow$(1): Let U be any μ-regular open subset of Y. Then U is μ-preopen. By (4), $f^{-1}(U) \subseteq m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(U)))) = m_X$ - $pInt(f^{-1}(U))$, and so $f^{-1}(U)$ is m_X-preopen. Hence, by Theorem 3.3.2(6), f is almost (m,μ)-precontinuous.

CHAPTER 4

WEAKY (m,μ)-PRECONTINUOUS FUNTIONS

In this section, we define the notion of weakly (m,μ)-precontinuous functions and investigate characterizations of weakly (m,μ)-precontinuous functions.

Definition 4.1 Let (X,m_X) be an m-space and (Y,μ) a generalized topological space. A function $f:(X,m_X)\to(Y,\mu)$ is said to be weakly (m,μ)-precontinuous at a point $x\in X$ if for each μ-open set V containing $f(x)$, there exists m_X-preopen U $U\in m_X$ containing x such that $f(U)\subseteq c_\mu(V)$. A function $f:(X,m_X)\to(Y,\mu)$ is said to be weakly (m,μ)-precontinuous if it has this property at each point $x\in X$.

Remark 4.2 From the above definitions. We have the following implication but the reverse relation may not be true in general:

almost (m,μ)-precontinuous$\Rightarrow$weakly (m,μ)-precontinuous.

Example 4.3 Let $X=\{1,2,3\}$, $m_X=\{\phi,\{3\},\{2,3\},X\}$ and $Y=\{a,b,c\}$, $\mu=\{\phi,\{b\},\{c\},\{b,c\},Y\}$ Define $f:(X,m_X)\to(Y,\mu)$ as follows: $f(1)=b$, $f(2)=a$ and $f(3)=c$ Then, f is weakly (m,μ)-precontinuous but it is not almost (m,μ)-precontinuous .

Theorem 4.4 A function $f:(X,m_X)\to(Y,\mu)$ is weakly (m,μ)-precontinuous at x if and only if for each μ-open set V containing $f(x)$, $x\in m_X\text{-}pInt(f^{-1}(c_\mu(V)))$.

Proof. Assume that f is weakly (m,μ)-precontinuous at x. Let V be an μ-open set containing $f(x)$. Thus, there exists an m_X-preopen set U containing x such that $f(U)\subseteq c_\mu(V)$. Then, $x\in U\subseteq f^{-1}(c_\mu(V))$, and so $x\in m_X\text{-}pInt(f^{-1}(c_\mu(V)))$.

Conversely, let $x\in X$ and V a μ-open set of Y containing $f(x)$. By assumption, we have $x\in m_X\text{-}pInt(f^{-1}(c_\mu(V)))$. Put $U=m_X\text{-}pInt(f^{-1}(c_\mu(V)))$. Then U is an m_X-preopen set in X containing x such that $f(U)\subseteq c_\mu(V)$.

Theorem 4.5 A function $f:(X,m_X)\rightarrow(Y,\mu)$ is weakly (m,μ)-precontinuous if and only if $f^{-1}(V)\subseteq m_X$ - $pInt(f^{-1}(c_\mu(V)))$ for every μ-open set V of Y.

Proof. Assume that f is weakly (m,μ)-precontinuous. Let V be an μ-open set of Y and $x\in f^{-1}(V)$. Then $f(x)\in V$. Since f is weakly (m,μ)-precontinuous at x, by Theorem 4.1, $x\in m_X$ - $pInt(f^{-1}(c_\mu(V)))$. Hence, $f(V)\subseteq m_X$ - $pInt(f^{-1}(c_\mu(V)))$.

Conversely, let $x\in X$ and let V be any μ-open set containing $f(x)$. By assumption, we have $x\in f^{-1}(V)\subseteq m_X$ - $pInt(f^{-1}(c_\mu(V)))$. By Theorem 4.1 f is weakly (m,μ)-precontinuous at x.

Theorem 4.6 For a function $f:(X,m_X)\rightarrow(Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-precontinuous;

(2) $f^{-1}(V)\subseteq m_X$ - $pInt(f^{-1}(c_\mu(V)))$ for every μ-open subset V of Y;

(3) m_X - $pCl(f^{-1}(i_\mu(F)))\subseteq f^{-1}(F)$ for every μ-closed subset F of Y;

(4) m_X - $pCl(f^{-1}(i_\mu(c_\mu(A))))\subseteq f^{-1}(c_\mu(A))$ for every subset A of Y;

(5) $f^{-1}(i_\mu(A))\subseteq m_X$ - $pInt(f^{-1}(c_\mu(i_\mu(A))))$ for every subset A of Y;

(6) m_X - $pCl(f^{-1}(V))\subseteq f^{-1}(c_\mu(V))$ for every μ-open subset V of Y.

Proof. (1)$\Rightarrow$(2): It follows from only if part of Theorem 4.2 .

(2)$\Rightarrow$(3): Let F be any μ-closed subset of Y. Then $Y-F$ is a μ-open subset of Y. By (2), we have

$$\begin{aligned} X - f^{-1}(F) &= f^{-1}(Y-F) \\ &\subseteq m_X\text{ - }pInt(f^{-1}(c_\mu(Y-F))) \\ &= m_X\text{ - }pInt(f^{-1}(Y-i_\mu(F))) \\ &= X - m_X\text{ - }pCl(f^{-1}(i_\mu(F))). \end{aligned}$$

Thus, $m_X\text{ - }pCl(f^{-1}(i_\mu(F)))\subseteq f^{-1}(F).$

(3)$\Rightarrow$(4): Let A be a subset of Y. Since $c_\mu(A)$ is a μ-closed in Y and by (3), it follows that $m_X\text{ - }pCl(f^{-1}(i_\mu(c_\mu(A))))\subseteq f^{-1}(c_\mu(A)).$

(4)$\Rightarrow$(5): Let A be a subset of Y. From (4), it follows that

$$\begin{aligned} f^{-1}(i_\mu(A)) &= X - f^{-1}(c_\mu(Y-A)) \\ &\subseteq X-(m_X\text{ - }pCl(f^{-1}(i_\mu(c_\mu(Y-A))))) \\ &= m_X\text{ - }pInt(f^{-1}(c_\mu(i_\mu(A)))). \end{aligned}$$

(5)$\Rightarrow$(6): Let V be an μ-open subset of Y. Suppose that $x\notin f^{-1}(c_\mu(V))$. Then $f(x)\notin c_\mu(V)$, and so there exists a μ-open set W containing $f(x)$ such that $V\cap W=\phi$. Thus $c_\mu(W)\cap V=\phi$. By (5), $x\in f^{-1}(W)\subseteq m_X\text{ - }pInt(f^{-1}(c_\mu(W)))$. Then there exists an m_X-preopen set G containing x such that $x\in G\subseteq f^{-1}(c_\mu(W))$. Since $c_\mu(W)\cap V=\phi$ and $f(G)\subseteq c_\mu(W)$, we have $G\cap f^{-1}(V)=\phi$. Thus, $x\notin m_X\text{ - }pCl(f^{-1}(V))$.
Hence $m_X\text{ - }pCl(f^{-1}(V))\subseteq f^{-1}(c_\mu(V))$.

(6) $\Rightarrow$ (1): Let $x\in X$ and let V be a μ-open subset of Y containing $f(x)$. By (6), we have

$$\begin{aligned} x\in f^{-1}(V) &\subseteq f^{-1}(i_\mu(c_\mu(V))) \\ &= X - f^{-1}(c_\mu(Y-c_\mu(V))) \\ &\subseteq X - m_X\text{ - }pCl\text{ - }(f^{-1}(Y-c_\mu(V))) \\ &= m_X\text{ - }pInt(f^{-1}(c_\mu(V))). \end{aligned}$$

Then there exists an m_X-preopen subset W of X containing x such that $W\subseteq f^{-1}(c_\mu(V))$. Hence, f is weakly (m,μ)- precontinuous.

Theorem 4.7 For a function $f:(X,m_X)\to(Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ) -precontinuous;

(2) m_X - $pCl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(F)$ for every μ-regular closed subset F of Y;

(3) m_X - $pCl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ - β-open subset G of Y;

(4) m_X - $pCl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-semi-open subset G of Y.

Proof. (1) $\Rightarrow$(2): Let F be any μ-regular closed subset of Y. Then $i_\mu(F)$ is μ-open, by Theorem 4.6 (6), we have m_X - $pCl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(c_\mu(i_\mu(F)))$. Since F is μ-regular closed, we have m_X - $pCl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(c_\mu(i_\mu(F))) = f^{-1}(F)$.

(2) $\Rightarrow$(3): Let G be any a μ - β-open subset of Y. Then $c_\mu(G) = c_\mu(i_\mu(c_\mu(G)))$ and so $c_\mu(G)$ is μ-regular closed. From (2), we have m_X - $pCl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$.

(3) $\Rightarrow$(4): It follows from fact that every μ -semi-open set is μ - β-open.

(4) $\Rightarrow$(1): Let V be any μ-open subset of Y. Then, by (4), m_X - $pCl(f^{-1}(V)) \subseteq m_X$ - $pCl(f^{-1}(i_\mu(c_\mu(V)))) \subseteq f^{-1}(c_\mu(V))$. Hence, By Theorem 4.6 (6), f is weakly (m,μ)-precontinuous.

Theorem 4.8 For a function $f:(X,m_X) \to (Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-precontinuous;

(2) m_X - $pCl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-preopen subset G of Y;

(3) m_X - $pCl(f^{-1}(G)) \subseteq f^{-1}(c_\mu(G))$ for every μ-preopen subset G of Y;

(4) $f^{-1}(G) \subseteq m_X$ - $pInt(f^{-1}(c_\mu(G)))$ of every μ-preopen subset G of Y;

Proof. (1) $\Rightarrow$(2): Let G be any μ-preopen subset of Y. Then $c_\mu(G) = c_\mu(i_\mu(c_\mu(G)))$, and so $c_\mu(G)$ is μ-regular closed. From Theorem 4.7 (2), it follows that m_X - $pCl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$.

(2) $\Rightarrow$(3): Let G be any μ-preopen subset of Y. Then $G \subseteq i_\mu(c_\mu(G))$. By (2), we have

m_X - $pCl(f^{-1}(G)) \subseteq m_X$ - $pCl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$.

(3) $\Rightarrow$(4): Let G be any μ-preopen subset of Y. By (3), we have

$$\begin{aligned} f^{-1}(G) &\subseteq f^{-1}(i_\mu(c_\mu(G))) \\ &= X - f^{-1}(Y - i_\mu(c_\mu(G))) \\ &= X - f^{-1}(c_\mu(Y - c_\mu(G))) \\ &\subseteq X - m_X\text{ - } pCl(f^{-1}(Y - c_\mu(G))) \\ &= m_X\text{ - } pInt(f^{-1}(c_\mu(G))). \end{aligned}$$

(4) $\Rightarrow$(1): Since every μ-open set is μ-preopen, by (4) and Theorem 4.6 (2), it follows that f is weakly (m, μ)-precontinuous.

CHAPTER 5

CONCLUSION

5.1 Conclusions

This thesis is aimed at studying the (m,μ)-precontinuous functions, almost (m,μ)-precontinuous functions and weakly (m,μ) precontinuous functions , First, We introduce the concept of above functions as follows:

(1) Let (X,m_X) be an m-space and (Y,μ) be a generalized topological space. A function $f:(X,m_X)\to(Y,\mu)$ is said to be (m,μ)- precontinuous at a point $x\in X$ if for each μ-open set V containing $f(x)$, there exists an m_X-preopen set U containing x such that $f(U)\subseteq V$. A function $f:(X,m_X)\to(Y,\mu)$ is said to be (m,μ)-precontinuous if f is (m,μ)-preontinuous at x for all $x\in X$.

(2) Let X be a nonempty set with m_X an m-structure on X. For a subset A of X the m_X-preclosure of A, denote by m_X- $pCl(A)$ and the m_X-preinterior of A, denote by m_X- $pInt(A)$ are defined as follow :

(2.1) m_X- $pCl(A)=\cap\{F : A\subseteq F, F$ is m_X-preclosed $\}$;

(2.2) m_X- $pCl(A)=\cup\{U : U\subseteq A, U$ is m_X-preopen$\}$.

From that definition 1 and 2, we derive attractive theorems as follows :

(I) Let X be nonempty set with m_X an m-structure on X. For a subset A of X, the following properties hold :

(3.1) m_X- $pCl(X-A)=X-(m_X$- $pInt(A))$:

(3.2) m_X- $pInt(X-A)=X-(m_X$- $pCl(A))$.

(II) Let X be nonempty set with m_X an m-structure on X. For a subset A of X, the following properties hold:

(1) A is m_X-preopen if and only if m_X- $pInt(A)=A$;

(2) A is m_X-preopen if and only if m_X- $pCl(A)=A$;

(3) m_X- $pInt$ $(m_X$- $pInt(A))=m_X$- $pInt(A)$;

(4) m_X - $pCl(m_X$ - $pCl(A)) = m_X$ - $pCl(A)$;

(5) m_X - $pInt(A)$ is m_X -preopen and m_X - $pCl(A)$ is m_X -preclosed.

(III) Let X be a nonempty set with a minimal structure m_X -preopen and a subset A of X. Then $x \in m_X$ - $pCl(A)$ if and only if $A \cap U \neq \phi$, U is m_X -preopen containing x.

(IV) For a function $f:(X, m_X) \to (Y, \mu)$ the following properties are equivalent:

(1) f is (m, μ) precontinuous at $x \in X$;

(2) $x \in m_X$ - $pInt(f^{-1}(V))$ for every $V \subseteq \mu$ containing $f(x)$;

(3) $x \in f^{-1}(c_\mu(f(A)))$ for every subset A of X with $x \in m_X$ - $pCl(A)$;

(4) $x \in f^{-1}(c_\mu(B))$ for every subset B of Y with $x \in m_X$ - $pCl(f^{-1}(B))$;

(5) $x \in m_X$ - $pInt(f^{-1}(B))$ for every subset B of Y with $x \in f^{-1}(i_\mu(B))$;

(6) $x \in f^{-1}(F)$ for every μ -closed subset F of Y such that $x \in m_X$ - $pCl(f^{-1}(F))$.

(V) For a function $f:(X, m_X) \to (Y, \mu)$, the following properties are equivalent:

(1) f is (m, μ) -precontinuous at $x \in X$;

(2) $f^{-1}(V)$ is m_X -preopen in X for every μ -open set V of Y;

(3) $f(m_X$ - $pCl(A) \subseteq c_\mu(f(A))$ for every subset A of X;

(4) m_X - $pCl(f^{-1}(B)) \subseteq f^{-1}(c_\mu(B))$ for every subset B of Y;

(5) $f^{-1}(i_\mu(B)) \subseteq m_X$ - $pInt(f^{-1}(B))$ for every subset B of Y;

(6) $f^{-1}(F)$ is m_X -preopen in X for every μ -closed set F of Y.

(3) Let (X, m_X) be an m-space and (Y, μ) be a generalized topological space. A function $f:(X, m_X) \to (Y, \mu)$ is said to be almost (m, μ)-precontinuous at a point $x \in X$ if for each μ-open set V containing $f(x)$, there exists a m_X-preopen set U containing x such that $f(U) \subseteq i_\mu(c_\mu(V))$. A function $f:(X, m_\mu) \to (Y, \mu)$ is said to be almost (m, μ)-precontinuous if f is almost (m, μ)-precontinuous at x for all $x \in X$. From that definition, we derive attractive theorems as follows:

(I) For a function $f:(X, m_X) \to (Y, \mu)$, the following properties are equivalent:

(1) f is almost (m, μ)-precontinuous at $x \in X$;

(2) $x \in m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open set V containing $f(x)$;

(3) $x \in m_X$ - $pInt(f^{-1}(V))$ for every μ-regular open set V containing $f(x)$;

(4) for every μ-regular open set V containing $f(x)$, there exists a m_X-preopen set U containing x such that $f(U) \subseteq V$.

(II) For a function $f:(X, m_X) \to (Y, \mu)$, the following properties are equivalent:

(1) f is almost (m, μ)-precontinuous ;

(2) $f^{-1}(V) \subseteq m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open set V of Y;

(3) m_X - $pCl(f^{-1}(c_\mu(i_\mu(F)))) \subseteq f^{-1}(F)$ for every μ-closed subset F of Y;

(4) m_X - $pCl(f^{-1}(c_\mu(i_\mu(c_\mu(B))))) \subseteq f^{-1}(c_\mu(B))$ for every subset B of Y;

(5) $f^{-1}(i_\mu(B)) \subseteq m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(i_\mu(B)))))$ for every subset B fo Y ;

(6) $f^{-1}(V)$ is m_X-preopen in X for every μ-regular open subset V of Y ;

(7) $f^{-1}(F)$ is m_X-preclosed in X for every μ-regular closed subset F of Y;

(III) For a function $f:(X,m_X)\to(Y,\mu)$, the following properties are equivalent:

(1) f is almost (m,μ)-precontinous;

(2) m_X- $pCl(f^{-1}(U))\subseteq f^{-1}(c_\mu(U))$ for every μ- β-open subset U of Y ;

(3) m_X- $pCl(f^{-1}(U))\subseteq f^{-1}(c_\mu(U))$ for every μ-preopen subset U of Y;

(4) $f^{-1}(U)\subseteq m_X$ - $pInt(f^{-1}(i_\mu(c_\mu(U))))$ for every μ-preopen subset U of Y.

(4) Let (X,m_X) be an m-space and (Y,μ) be a generalized topological space. A function $f:(X,m_X)\to(Y,\mu)$ is said to be weakly (m,μ)-precontinuous at a point $x\in X$ if for each μ-open set V containing $f(x)$, there exists a m_X-preopen set U containing x such that $f(U)\subseteq c_\mu(V)$. A function $f:(X,m_X)\to(Y,\mu)$ is said to be weakly (m,μ)-precontinuous if f is weakly (m,μ)-precontinuous at x for all $x\in X$.

(I). A function $f:(X,m_X)\to(Y,\mu)$ is weakly (m,μ)-precontinuous at x if and only if for each μ-open set V containing $f(x)$, $x\in m_X$ - $pInt(f^{-1}(c_\mu(V)))$.

(II). A function $f:(X,m_X)\to(Y,\mu)$ is weakly (m,μ)-precontinuous if and only if $f^{-1}(V)\subseteq m_X$ - $pInt(f^{-1}(c_\mu(V)))$ for every μ-open set V of Y .

(III). For a function $f:(X,m_X)\to(Y,\mu)$, the following properties are equivalent :

(1) f is weakly (m,μ)-precontinuous;

(2) $f^{-1}(V)\subseteq m_X$ - $pInt(f^{-1}(c_\mu(V)))$ for every μ-open subset V of Y;

(3) m_X - $pCl(f^{-1}(i_\mu(F)))\subseteq f^{-1}(F)$ for every μ-closed subset F of Y;

(4) m_X - $pCl(f^{-1}(i_\mu(c_\mu(A)))) \subseteq f^{-1}(c_\mu(A))$ for every subset A of Y ;

(5) $f^{-1}(i_\mu(A)) \subseteq m_X$ - $pInt(f^{-1}(c_\mu(i_\mu(A))))$ for every subset A of Y ;

(6) m_X - $pCl(f^{-1}(V)) \subseteq f^{-1}(c_\mu(V))$ for every μ-open subset V of Y

(IV). For a fuction $f:(X,m_X) \to (Y,\mu)$, the following properties are equivalent:

(1) f is weakly (X,μ)-precontinuous;

(2) m_X - $pCl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(F)$ for every μ-regular closed subset F of Y ;

(3) m_X - $pCl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ - β-open subset G of Y ;

(4) m_X - $pCl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-preopen subset G of Y ;

(V) For a fuction $f:(X,m_X) \to (Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-precontinuous;

(2) m_X - $pCl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-preopen subset G of Y ;

(3) m_X - $PCl(f^{-1}(G)) \subseteq f^{-1}(c_\mu(G))$ for every μ-preopen subset G of Y ;

(4) $f^{-1}(G) \subseteq m_X$ - $pInt(f^{-1}(c_\mu(G)))$ for every μ-preopen subset G of Y ;

We have finally discovered that certain relationship of (m,μ)-precontinuous function, almost (m,μ)-precontinuous function, and weakly (m,μ)-precontinuous function as follows that (m,μ)-precontinuous function always imply almost (m,μ)-precontinuous function and almost (m,μ)-precontinuous function, we have a following weakly (m,μ)-precontinuous function implications but the reverse relations may not be true in general:

(m,μ)-precontinuous function

$\Downarrow$

almost (m,μ)-precontinuous function

$\Downarrow$

weakly (m,μ)-precontinuous function

5.2 Recommendations

To this end, even though I have found several properties as presented in this thesis, there are several questions yet to be answered and it may be worth investigating in future studies. I formulate the questions as follows:

1. Are there any properties of (m,μ)-precontinuous function, almost (m,μ)-precontinuous function and weakly (m,μ)-precontinuous function ?

2. Is there any property of these functions on other structure space?

3. Do these functions have any connections with others?

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BIOGRAPHY

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