(m, μ)-CONTINUOUS FUNCTIONS

SUPAWIMOL PHOLDEE

A thesis submitted in partial fulfillment of the requirements for
the degree of Master of Science in Mathematics Education
Mahasarakham University
August 2012

(m, μ)-CONTINUOUS FUNCTIONS

SUPAWIMOL PHOLDEE

A thesis submitted in partial fulfillment of the requirements for
the degree of Master of Science in Mathematics Education
Mahasarakham University
August 2012

The examining committee has unanimously approved this thesis, submitted by Miss. Supawimol Pholdee, as a partial fulfillment of the requirements for the Master of Science degree in Mathematics Education, Mahasarakham University.

Examining Committee

Maliwan Tu .... Chairman

(Asst.Prof. Maliwan Tunapan, Ph.D) (Faculty graduate committee)

C. Boonpok .... Committee

(Asst.Prof. Chawalit Boonpok, Ph.D.) (Advisor)

C. Viriyapong .... Committee

(Chokchai Viriyapong, Ph.D.) (Co - advisor)

.... Committee

(Ekjittra Srisarakham, Ph.D.) (Faculty graduate committee)

.... Committee

(Supunnee Sanpinij, Ph.D.) (External expert)

Mahasarakham University has granted approval to accept this thesis as a partial fulfillment of the requirements for the Master of Science degree in Mathematics Education.

La-orsri Sanoamuang ....

(Prof. La-orsri Sanoamuang, Ph.D.)

Dean of the Faculty of Science

N. T ....

(Assoc.Prof. Nathanon Trachoo, Ph.D.)

Acting Dean of the Faculty of Graduate Studies

August 29, 2012

ชื่อเรื่อง ฟังก์ชันต่อเนื่อง (m,μ)
ผู้วิจัย นางสาวศุภวิมล ผลดี
ปริญญา วิทยาศาสตรมหาบัณฑิต **สาขาวิชา** คณิตศาสตรศึกษา
กรรมการควบคุม ผู้ช่วยศาสตราจารย์ ดร.ชวลิต บุญปก
ดร.โชคชัย วิริยะพงษ์
มหาวิทยาลัย มหาวิทยาลัยมหาสารคาม **ปีที่พิมพ์** 2555

บทคัดย่อ

งานวิจัยนี้ผู้วิจัยได้นิยามฟังก์ชันต่อเนื่อง(m,μ) ฟังก์ชันเกือบต่อเนื่อง(m,μ)และฟังก์ชันต่อเนื่องอย่างอ่อน(m,μ) ซึ่งเป็นฟังก์ชันจากปริภูมิโครงสร้างเล็กสุดไปยังปริภูมิเชิงทอพอโลยีวางนัยทั่วไป จากการศึกษาได้สมบัติบางอย่าง ลักษณะเฉพาะและความสัมพันธ์ระหว่างฟังก์ชันต่อเนื่อ(m,μ) ฟังก์ชันเกือบต่อเนื่อง (m,μ) และฟังก์ชันต่อเนื่องอย่างอ่อน (m,μ)

คำสำคัญ : ปริภูมิโครงสร้างเล็กสุด ; ปริภูมิเชิงทอพอโลยีวางนัยทั่วไป ; ฟังก์ชันต่อเนื่อง(m,μ) ; ฟังก์ชันเกือบต่อเนื่อง (m,μ) ; ฟังก์ชันต่อเนื่องอย่างอ่อน (m,μ)

TITLE	(m,μ)-continuous functions
CANDIDATE	Miss Supawimol Pholdee
DEGREE	Master Degree of Science **MAJOR** Mathematics Education.
ADVISORS	Asst. Prof. Chawalit Boonpok, Ph.D. Chokchai Viriyapong, Ph.D.
UNIVERSITY	Mahasarakham University **DATE** 2012

ABSTRACT

In this research, we introduce the notion of (m,μ)-continuous functions, almost (m,μ)-continuous functions and weakly (m,μ)-continuous functions as functions from an m-space into a generalized topological space. Some properties, characterizations and relationships between (m,μ)-continuous functions, almost (m,μ)-continuous functions and weakly (m,μ)-continuous functions are obtained.

Keywords : minimal structure space ; generalized topological space ; (m,μ)-continuous functions ; almost (m,μ)-continuous functions ; weakly (m,μ)-continuous functions.

ACKNOWLEGDEMENTS

I wish to express my deepest sincere gratitude to Asst. Prof. Dr.Chawalit Boonpok and Dr. Chokchai Viriyapong for their initial idea, guidance and encouragement which enable me to carry out my study successfully.

I would like to thank Asst. Prof. Dr. Maliwan Tunapan, Dr.Supunnee Sanpinij and Dr.Aekjittra Srisarakham for their constructive comments and suggestions.

I extent my thanks to all my teachers for their previous lectures.

I would like to express my sincere gratitude to my beloved parents, Mr.Thaem Pholdee and Mrs.Supaporn Pholdee, my sister, Miss Phattaraporn Pholdee, my friends and my students who continuously encourage me.

I would like to thank all graduate students and staffs at the Department of Mathematics for supporting on this preparation of the thesis.

Finally, I would like to thank Mahasarakham University for financial support of the thesis.

Supawimol Pholdee

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Background

In the current, the mathematician interested in topology to construct and study the properties. Topology is the one of main branch of mathematics which interested in the properties that are invariant under a change in the contraction force and preserved under continuous deformations of objects, such as deformations that involve stretching, but no tearing or gluing. It emerged through the development of concepts from geometry and set theory, such as space, dimension, and transformation.

Topological spaces are mathematical structures that allow the formal definition of concepts such as convergence, connectedness, and continuity. They appear in virtually every branch of modern mathematics and are a central unifying notion. The branch of mathematics that studies topological spaces in their own right is called topology.

The concept of generalized neighborhood systems and generalized topology space were introduced by Á. Császár [1] in order to explain interior and operation of closure in neighborhood systems and generalized topology space. K. M. Min [6] studied some notion of continuous function on generalized topology space. Further, Á. Császár [2] studied and defined μ-α-open set, μ-semi-open set, μ-preopen set and μ-β-open set on generalized topology space.

In 2000 Pupa and Noiri [7] studied and defined the notion of minimal structure that let $X \neq \phi$ and $P(X)$ is power set of X, we called $m \subseteq P(X)$ that is minimal structure (briefly m-structure) on X if only if $\phi \in m$ and $X \in m$. So (X,m) is called m-space. Each element of m is said to be m-open and complement of an m-open set is said to be m-closed. There from, C.Boonpok [3, 4] introduced and studied almost (g,m)-continuous functions and weakly (g,m)-continuous functions on generalized topology space. He also studied some properties that functions.

The thesis is divided into five chapters. The first chapter comprises an introduction which contains some historical remark concerning the research specialization. We also explain our motivations and outline the goals of the thesis here. In the second chapter, we give some definitions, notations and some know theorems that will be used in the later chapters. In the third and the fourth chapter, we give some definitions, notations and some interesting proposition of (m,μ)-continuous functions, almost (m,μ)-continuous functions and weakly (m,μ)-continuous functions, respective as functions from an m-space into a generalized topological space which is the fundamental properties for the last chapter and we give their characterization. In the last chapter, we draw conclusions based on the obtained results and outline a possible direction foe further research.

1.2 Objective of the research

The purposes of the research are:

1.2.1 To construct and investigate the notion and properties of (m,μ)-continuous functions.

1.2.2 To construct and study the properties of almost (m,μ)-continuous functions.

1.2.3 To construct and study the properties of weakly (m,μ)-continuous functions.

1.2.4 To study the relation of properties among (m,μ)-continuous functions, almost (m,μ)-continuous functions and weakly (m,μ)-continuous functions.

1.3 Research methodology

The research procedure of this thesis consists of the following step:

1.3.1 Criticism and possible extensions of the literature review.

1.3.2 Doing research to investigate the main results.

1.3.3 Applying the results from 1.3.1and 1.3.2 to main results.

1.3.4 Making the conclusions and recommendations.

1.4 Scope of the study

The scopes of the study are:

1.4.1 Construct and investigate the notion and properties of (m,μ)-continuous functions, almost (m,μ)-continuous functions and weakly (m,μ)-continuous functions.

1.4.2 Study the relation of properties among (m, μ)-continuous functions, almost (m, μ)-continuous functions and weakly (m, μ)-continuous functions.

CHAPTER 2

PRELIMINARIES

This chapter comprises the fundamental properties needed in the proof of each theorem in the study.

2.1 Generalized Topology Spaces

In this section, we give some definitions, notations and some known propositions of generalized topological spaces that will be used in the later chapter.

Definition 2.1.1 [1] Let X be a nonempty set and μ a collection of subsets of X. Then μ is called a generalized topology (briefly GT) on X if and only if $\phi \in \mu$ and $\bigcup_{i\in I}\mu_i \in \mu$ for $i \in I \neq \phi$ by that $\mu_i \in \mu$. We call the pair (X,μ) a generalized topological space (briefly GTS) on X. The element of μ are called μ-open and the complements are called μ-closed sets.

The closure of a subset A in a generalized topology space (Y,μ) denoted by $c_\mu(A)$ and the interior of subset A denoted $i_\mu(A)$.

Theorem 2.1.2 [1] Let $X \neq \phi$ and μ be generalized topological space and $A \subseteq Y$. Then

(1) $C_\mu(A) = \bigcap\{F \mid X - F \in \mu, A \subseteq F\}$

(2) $i_\mu(A) = \bigcup\{G \subseteq A \mid G \in \mu\}$

Theorem 2.1.3 [1] Let (Y,μ) be generalized topological space and $A \subseteq Y$. Then

(1) $c_\mu(A) = Y - i_\mu(Y - A)$

(2) $i_\mu(A) = Y - c_\mu(Y - A)$

$gO(Y) = \{U \subseteq Y : U \in \mu\}$ and $gO(y) = \{U \in \mu : y \in U\}$.

Proposition 2.1.4 [3] Let (Y,μ) be generalized topological space and $A \subseteq Y$. Then

(1) $y \in i_{\mu}(A)$ if only if there exists $V \in gO(Y)$ such that $V \subseteq A$.

(2) $y \in c_{\mu}(A)$ if only if $V \cap A \neq \phi$ for every $V \in gO(y)$.

Definition 2.1.5 [2] Let (Y,μ) be generalized topological space and $A \subseteq Y$ is said to be :

(1) μ - semiopen set if $A \subseteq c_{\mu}(i_{\mu}(A))$

(2) μ - preopen set if $A \subseteq i_{\mu}(c_{\mu}(A))$

(3) μr - open set if $A = i_{\mu}(c_{\mu}(A))$

(4) μ-β- open set if $A \subseteq c_{\mu}(i_{\mu}(c_{\mu}(A)))$

The complement of μ-semiopen (resp. μ-preopen, μr-open μ-β-open) set is called μ-semiclosed (μ-preclosed , μr-closed , μ-β-closed).

2.2 Minimal Structure Spaces

Definition 2.2.1 [7] Let $X \neq \phi$ and $P(X)$ be the power set of X. $m_x \subseteq P(X)$ is said to be minimal structure on X if only if $\phi \in m_x$ and $X \in m_x$. (X, m_x) is said to be *m*-space and each member of *m* is said to be m-open set. The complement of *m*-open set is called m-closed set

Definition 2.2.2 [6] Let $X \neq \phi$ and m_x be minimal structure on X. m is said to have property B if only if the union of subsets belonging to m_x belongs to m_x.

Definition 2.2.3 [7] Let $X \neq \phi$ and m_x be minimal structure on X. For each $A \subseteq X$ m_x-closure of A and m_x-interior of A. The following definition hold:

(1) $m\text{-}Cl(A) = \bigcap\{F : A \subseteq F, X - F \in m_x\}$

(2) $m\text{-}Int(A) = \bigcup\{U : U \subseteq A, U \in m_x\}$

Lemma 2.2.4 [5] Let $X \neq \phi$ and m_x be minimal structure on X satisfying property B. For each $A \subseteq X$ the following properties hold:

(1) $A \in m_x$ if only if $m\text{-}Int(A) = A$;

(2) A be m-closed set if only if $m\text{-}Cl(A) = A$;

(3) $m\text{-}Int(A) \in m_x$ and $m\text{-}Cl(A)$ be m-closed set.

Lemma 2.2.5 [7] Let $X \neq \phi$ and m_x be minimal structure on X. For $A, B \subseteq X$ the following properties hold:

(1) $m\text{-}Cl(X - A) = X - m\text{-}Int(A)$ and $m\text{-}Int(X - A) = X - m\text{-}Cl(A)$;

(2) if $(X - A) \in m_x$ then $m\text{-}Cl(A) = A$ and if $A \in m_x$ then $m\text{-}Int(A) = A$;

(3) $m\text{-}Cl(\phi) = \phi$, $m\text{-}Cl(X) = X$, $m\text{-}Int(\phi) = \phi$ and $m\text{-}Int(X) = X$;

(4) if $A \subseteq B$ then $m\text{-}Cl(A) \subseteq m\text{-}Cl(B)$ and $m\text{-}Int(A) \subseteq m\text{-}Int(B)$;

(5) $A \subseteq m\text{-}Cl(A)$ and $m\text{-}Int(A) \subseteq A$;

(6) $m\text{-}Cl(m\text{-}Cl(A)) = m\text{-}Cl(A)$ and $m\text{-}Int(m\text{-}Int(A)) = m\text{-}Int(A)$.

2.3 (g,m)- Continuous Functions

Definition 2.3.1 [3] A function $f:(X,g_x) \to (Y,m_Y)$ is said to be (g,m)-continuous at a point $x \in X$ if for each m-open set V containing $f(x)$, there exists a g_x-open set U containing x such that $f(U) \subseteq V$. A function $f:(X,g_x) \to (Y,m_Y)$ is said to be (g,m)-continuous if it has this property at each point $x \in X$.

Theorem 2.3.2 [3] For a function $f:(X,g_x) \to (Y,m_Y)$, the following properties are equivalent:

(1) f is (g,m)-continuous at $x \in X$;

(2) $x \in i_x(f^{-1}(V))$ for every $V \in m_Y$ containing $f(x)$;

(3) $x \in f^{-1}(m_Y\text{-}Cl(f(A)))$ for every subset A of X with $x \in c_x(A)$;

(4) $x \in f^{-1}(m\text{-}Cl(f(B))$ for every subset B of Y with $x \in c_x(f^{-1}(B))$;

(5) $x \in i_x(f^{-1}(B))$ for every subset B of Y with $x \in f^{-1}(m_Y\text{-}Int(B))$;

(6) $x \in f^{-1}(K)$ for every subset m- closed set K of Y such that

$x \in c_x(f^{-1}(K))$.

Definition 2.3.3 [3] A function $f:(X,g_x)\rightarrow(Y,m_Y)$ is said to be almost (g,m)-continuous at a point $x\in X$ if for each m_Y-open set V containing $f(x)$, there exists a g_x-open set U containing x such that $f(U)\subseteq m_Y\text{-}Int(m_Y\text{-}Cl(V))$. A function $f:(X,g_x)\rightarrow(Y,m_Y)$ is said to be almost (g,m)-continuous if it has this property at each point $x\in X$.

Remark 2.3.4 [3] From the above definitions of (g,m)-continuity and almost (g,m)-continuity, we have the following implication but the reverse relation may not be true in general:

$$(g,m)\text{-continuous} \Longrightarrow \text{almost } (g,m)\text{-continuous}$$

Definition 2.3.4 [3] A subset A of a m-space (X,m_x) is said to be

(1) m_x-regular open if $A=m_x\text{-}Int(m_x\text{-}Cl(A))$

(2) m_x-semi-open if $A\subseteq m_x\text{-}Cl(m_x\text{-}Int(A))$

(3) m_x-preopen if $A\subseteq m_x\text{-}Int(m_x\text{-}Cl(A))$

(4) m_x-α-open if $A\subseteq m_x\text{-}Int(m_x\text{-}Cl(m_x\text{-}Int(A)))$

(5) m_x-β-open if $A\subseteq m_x\text{-}Cl(m_x\text{-}Int(m_x\text{-}Cl(A))$

The complement of a m_x-regular open (resp. m_x-semi-open, m_x-preopen, m_x-α-open, m_x-β-open) set is called m_x-regular closed (resp. m_x-semi-closed, m_x-preclosed, m_x-α-closed, m_x-β-closed) set.

Lemma 2.3.5 [3] Let (X,m_x) be a m-space and A a subset of X.

(1) A is m_x-regular closed if only if $A=m_x\text{-}Cl(m_x\text{-}Int(A))$

(2) A is m_x-semi- closed if only if $m_x\text{-}Int(m_x\text{-}Cl(A))\subseteq A$

(3) A is m_x-preclosed if only if $m_x\text{-}Cl(m_x\text{-}Int(A))\subseteq A$

(4) A is m_x-α-closed if only if $m_x\text{-}Int(m_x\text{-}Cl(m_x\text{-}Int(A)))\subseteq A$

Theorem 2.3.6 [3] For a function $f:(X,g_x)\rightarrow(Y,m_Y)$, the following properties are equivalent:

(1) f is almost (g,m)-continuous at $x\in X$;

(2) $x \in i_x(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(V))))$ for every m_Y-open set V containing $f(x)$;

(3) $x \in i_x(f^{-1}(V))$ for every m_Y- regular open set V containing $f(x)$;

(4) For every m_Y- regular open set V containing $f(x)$, there exists a g_x- open set U containing x such that $f(U) \subseteq V$.

Theorem 2.3.7 [3] For a function $f:(X,g_x) \longrightarrow (Y,m_Y)$ which m_Y has property B , the following properties are equivalent:

(1) f is almost (g,m)-continuous;

(2) $f^{-1}(V) \subseteq i_x(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(V))))$ for every m_Y-open subset V of Y;

(3) $c_x(f^{-1}(m_Y\text{-}Cl(m_Y\text{-}Int(F)))) \subseteq f^{-1}(F)$ for every m_Y-closed subset F of Y;

(4) $c_x(f^{-1}(m_Y\text{-}Cl(m_Y\text{-}Int(m_Y\text{-}Cl(B))))) \subseteq f^{-1}(m_Y\text{-}Cl(B))$ for every subset B of Y;

(5) $f^{-1}(m_Y\text{-}Int(B)) \subseteq i_x(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(m_Y\text{-}Int(B)))))$ for every subset B of Y;

(6) $f^{-1}(V)$ is g_x-open in X for every m_Y- regular open subset V of Y;

(7) $f^{-1}(F)$ is g_x-closed in X for every m_Y- regular closed subset V of Y.

Theorem 2.3.8 [3] For a function $f:(X,g_x) \longrightarrow (Y,m_Y)$, the following properties are equivalent:

(1) f is almost (g,m)-continuous;

(2) $c_x(f^{-1}(U)) \subseteq f^{-1}(m_Y\text{-}Cl(U))$ for every m_Y-β open subset U of Y;

(3) $c_x(f^{-1}(U)) \subseteq f^{-1}(m_Y\text{-}Cl(U))$ for every m_Y-semi open subset U of Y;

(4) $f^{-1}(U) \subseteq i_x(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(U))))$ for every m_Y-preopen subset U of Y.

Definition 2.3.9 [4] A function $f:(X,g_x) \longrightarrow (Y,m_Y)$ is said to be weakly (g,m)- continuous at a point $x \in X$ if for each m_Y- open set V containing $f(x)$, there exists a g_x-open set U containing x such that $f(U) \subseteq m_Y\text{-}Cl(V)$. A function $f:(X,g_x) \longrightarrow (Y,m_Y)$ is said to be weakly (g,m)-continuous if it has this property at each point $x \in X$.

Remark 2.3.10 [4] From the above definitions of (g,m)-continuity and weakly (g,m)-continuity, we have the following implication but the reverse relation may not be true in general:

(g,m)-continuous $\Rightarrow$ weakly (g,m)-continuous

Theorem 2.3.11 [4] A function $f:(X,g_X)\rightarrow(Y,m_Y)$ is weakly (g,m)-continuous at x if only if each m_Y-open set V containing $f(X)$, $x\in i_X(f^{-1}(m_Y\text{-}Cl(V)))$

Theorem 2.3.12 [4] A function $f:(X,g_X)\rightarrow(Y,m_Y)$ is weakly (g,m)-continuous if only if $f^{-1}(V)\subseteq i_X(f^{-1}(m_Y\text{-}Cl(V)))$ for every m_Y-open set V of Y.

Theorem 2.3.13 [4] For a function $f:(X,g_X)\rightarrow(Y,m_Y)$ which m_Y has property B , the following properties are equivalent:

(1) f is weakly (g,m)-continuous;

(2) $f^{-1}(U)\subseteq i_X(f^{-1}(m_Y\text{-}Cl(U)))$ for every m_Y-open subset V of Y;

(3) $c_X(f^{-1}(m_Y\text{-}Int(F)))\subseteq f^{-1}(F)$ for every m_Y-closed subset F of Y;

(4) $c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(A))))\subseteq f^{-1}(m_Y\text{-}Cl(A))$ for every subset A of Y;

(5) $f^{-1}(m_Y\text{-}Int(A))\subseteq i_X(f^{-1}(m_Y\text{-}Cl(m_Y\text{-}Int(A))))$ for every subset A of Y;

(6) $c_X(f^{-1}(U))\subseteq f^{-1}(m_Y\text{-}Cl(U))$ for every m_Y-open subset A of Y.

Theorem 2.3.14 [4] For a function $f:(X,g_X)\rightarrow(Y,m_Y)$, the following properties are equivalent:

(1) f is weakly (g,m)-continuous;

(2) $c_X(f^{-1}(m_Y\text{-}Int(F)))\subseteq f^{-1}(F)$ for every m_Y- regular subset F of Y;

(3) $c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G))))\subseteq f^{-1}(m_Y\text{-}Cl(G))$ for every m_Y-β-open subset G of Y;

(4) $c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G))))\subseteq f^{-1}(m_Y\text{-}Cl(G))$ for every m_Y-semi open subset G of Y.

Theorem 2.3.15 [4] For a function $f:(X,g_X)\to(Y,m_Y)$, the following properties are equivalent:

(1) f is weakly (g,m)-continuous;

(2) $c_X(f^{-1}(m_Y\text{-}Int(m_Y\text{-}Cl(G))))\subseteq f^{-1}(m_Y\text{-}Cl(G))$ for every m_Y- preopen subset G of Y;

(3) $c_X(f^{-1}(G))\subseteq f^{-1}(m_Y\text{-}Cl(G))$ for every m_Y- preopen subset G of Y;

(4) $f^{-1}(G)\subseteq i_X(f^{-1}(m_Y\text{-}Cl(G)))$ for every m_Y- preopen subset G of Y.

CHAPTER 3

(m,μ) - CONTINUOUS FUNCTIONS

3.1 (m,μ) - Continuous Functions

In this section, we introduce the concept of (m,μ)-continuous functions and investigate some of their characterizations.

Definition 3.1.1 Let (X,m_x) be a minimal structure space and (Y,μ) be a generalized topological space. A function $f:(X,m_x)\longrightarrow(Y,\mu)$ is said to be (m,μ)-continuous at a point $x\in X$ if for each μ-open set V containing $f(x)$, there exists a m_x-open set U containing x such that $f(U)\subseteq V$. A function $f:(X,m_x)\longrightarrow(Y,\mu)$ is said to be (m,μ)-continuous if it has this property at each point $x\in X$.

Example 3.1.2 Let X = {1, 2, 3}, m_X = {$\emptyset$, {1}, {2}, {1, 2}, X} , Y ={a, b, c} and μ ={$\emptyset$, {a}, {a, b}, Y} . Define $f:(X,m_x)\longrightarrow(Y,\mu)$ as follows : f(1)=a, f(2)=b and f(3)=c. Then f is (m,μ)-continuous.

Theorem 3.1.3 Let (X,m_x) be a minimal structure space and (Y,μ) be a generalized topological space. For a function $f:(X,m_x)\longrightarrow(Y,\mu)$, the following properties are equivalent:

(1) f is (m,μ)-continuous at $x\in X$;

(2) $x\in m_x\text{-}Int(f^{-1}(V))$ for every $V\in\mu$ containing $f(x)$;

(3) $x\in f^{-1}(c_\mu(f(A)))$ for every subset A of X with $x\in m_x\text{-}Cl(A)$;

(4) $x\in f^{-1}(c_\mu(B))$ for every subset B of Y with $x\in m_x\text{-}Cl(f^{-1}(B))$;

(5) $x\in m_x\text{-}Int(f^{-1}(B))$ for every subset B of Y with $x\in f^{-1}(i_\mu(B))$;

(6) $x\in f^{-1}(F)$ for every μ-closed set F of Y such that $x\in m_x\text{-}Cl(f^{-1}(F))$.

Proof. (1)$\to$(2) Let V be any μ-open subset Y containing $f(x)$ .Then, there exists a m_x-open subset U containing x such that $f(U)\subseteq V$. Since $U\in m_x$, we have $x\in m_x$-$Int(f^{-1}(V))$.

(2) $\to$(3) Let A be any subset X, $x\in m_x$-$Cl(A)$ and $V\in\mu$ containing $f(x)$. Then $x\in m_x$-$Int(f^{-1}(V))$. There exists $U\in m_x$ such that $x\in U\subseteq m_x$-$Int(f^{-1}(V))$.
Since $x\in m_x$-$Cl(A)$, $U\cap A\neq\phi$, by Proposition 2.1.4 and $\phi\neq f(U\cap A)\subseteq f(U)\cap f(A)$ $\subseteq V\cap f(A)$.Since $V\in\mu$ containing $f(x)$, $f(x)\in c_\mu(f(A))$ and hence $x\in f^{-1}(c_\mu(f(A)))$.

(3) $\to$(4) Let B be a any subset of Y and $x\in m_x$-$Cl(f^{-1}(B))$. By (3), $x\in f^{-1}(c_\mu(f(f^{-1}(B))))\subseteq f^{-1}(c_\mu(B))$. Hence, we have $x\in f^{-1}(c_\mu(B))$.

(4) $\to$(5) Let B be a any subset of Y such that $x\notin m_x$-$Int(f^{-1}(B))$. Then $x\in X-m_x$-$Int(f^{-1}(B))=m_x$-$Cl(X-f^{-1}(B))=m_x$-$Cl(f^{-1}(Y-B))$. By (4) , we have $x\in f^{-1}(c_\mu(Y-B))=f^{-1}(Y-(i_\mu(B)))=X-f^{-1}(i_\mu(B))$. Hence $x\notin f^{-1}(i_\mu(B))$.

(5) $\to$(6) Let F be any μ-closed set of Y such that $x\notin f^{-1}(F)$. then $x\in X-f^{-1}(F)=f^{-1}(Y-F)=f^{-1}(i_\mu(Y-F))$ because $Y-F$ is μ-open. By (5) , we have $x\in m_x$-$Int(f^{-1}(Y-F))=m_x$-$Int(X-f^{-1}(F))=X-m_x$-$Cl(f^{-1}(F))$. Hence, $x\notin m_x$-$Cl(f^{-1}(F))$.

(6) $\to$(2) Let $x\in X$ and $V\in\mu$ containing $f(x)$. Suppose that $x\notin m_x$-$Int(f^{-1}(V))$. Then $x\in X-m_x$-$Int(f^{-1}(V))=m_x$-$Cl(X-f^{-1}(V))$ $=m_x$-$Cl(f^{-1}(Y-V))$ .By (6), $x\in f^{-1}(Y-V)=X-f^{-1}(V)$. Hence $x\notin f^{-1}(V)$. This contraries to the hypothesis.

(2) $\to$(1) Let $V\in\mu$ containing $f(x)$. By (2), $x\in m_x$-$Int(f^{-1}(V))$ and hence there exists $U\in m_x$ containing x such that $x\in U\subseteq f^{-1}(V)$. Therefore, $f(U)\subseteq V$ and f is (m,μ)-continuous at x.

3.2 Almost (m, μ)-Continuous Functions

In this section, we define and the notion of almost (m, μ)-continuous functions and investigate some of their characterizations.

Definition 3.2.1 Let (X, m_x) be a minimal structure space and (Y, μ) be a generalized topological space. A function $f : (X, m_x) \to (Y, \mu)$ is said to be almost (m, μ)-continuous at a point $x \in X$ if for each μ-open set V containing $f(x)$, there exists a m_x-open set U containing x such that $f(U) \subseteq i_\mu(c_\mu(V))$. A function $f : (X, m_x) \to (Y, \mu)$ is said to be almost (m, μ)-continuous if it has this property at each point $x \in X$.

Example 3.2.2 Let X = {1, 2, 3}, m_x = {$\emptyset$, {1}, {2}, {1, 2}, X} , Y = {a, b, c}and μ = {$\emptyset$, {a}, {a, b}, Y}. Define $f : (X, m_x) \to (Y, \mu)$ as follows : f(1)=a, f(2)=b and f(3)=c. Then f is almost (m, μ)-continuous.

Remark 3.2.3 From the above definition of (m, μ)-continuity and almost (m, μ)-continuity, we have a following implication but the reverse relation may not be true in general:

$$(m, \mu)\text{-continuous} \Rightarrow \text{almost } (m, \mu)\text{-continuous}$$

Example 3.2.4 Let X = {1, 2}, m_x = {$\emptyset$, {1}, X } , Y = {a, b}, μ ={$\emptyset$, Y}.Define $f : (X, m_x) \to (Y, \mu)$ as follows: f(1) = a and f(2) = b. Then f is almost (m, μ)-continuous but it is not (m, μ)-continuous.

Theorem 3.2.5 Let (X, m_x) be a minimal structure space and (Y, μ) be a generalized topological space. For a function $f : (X, m_x) \to (Y, \mu)$, the following properties are equivalent:

(1) f is almost (m, μ)-continuous at $x \in X$;

(2) $x \in m_x\text{-}Int(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open set V containing $f(x)$;

(3) $x \in m_x\text{-}Int(f^{-1}(V))$ for every μ-regular open set V containing $f(x)$;

(4) For every μ- regular open set V containing $f(x)$, there exists a μ-open set U containing x such that $f(U) \subseteq V$.

Proof. (1) $\rightarrow$(2) Let V be any μ-open subset of Y containing $f(x)$. Then, there exists a m_x-open subset U of X containing x such that $f(U) \subseteq i_\mu(c_\mu(V))$. Then $x \in U \subseteq f^{-1}(i_\mu(c_\mu(V)))$. Since $U \in m_x$, we have $x \in m_x$-$Int(f(i_\mu(c_\mu(V))))$.

(2) $\rightarrow$(3) Let V be any μ- regular open subset of Y containing $f(x)$.Then $V = i_\mu(c_\mu(V))$, By (2) , we have $x \in m_x$-$Int(f^{-1}(V))$.

(3) $\rightarrow$(4) Let V be any μ- regular open subset of Y containing $f(x)$. By (3), we have a m_x-open set U of X containing x such that $U \subseteq f^{-1}(V)$. Hence $f(U) \subseteq V$.

(4) $\rightarrow$(1) Let $x \in X$ and let V be any μ-open subset Y containing $f(x)$.Then $f(x) \in V \subseteq i_\mu(c_\mu(V))$. Since $i_\mu(c_\mu(V))$ is μ- regular open, By (4),we have a m_x-open set U of X containing x such that $f(U) \subseteq i_\mu(c_\mu(V))$.

Hence, f is almost (m,μ)-continuous at $x \in X$.

Theorem 3.2.6 Let (X, m_x) be a minimal structure space and (Y, μ) be a generalized topological space. For a function $f:(X, m_x) \rightarrow (Y, \mu)$ which m_x has property B , the following properties are equivalent:

(1) f is almost (m,μ)-continuous;

(2) $f^{-1}(V) \subseteq m_x$ -$Int(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open subset V of Y ;

(3) m_x-$Cl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(F)$ for every μ-closed subset F of Y ;

(4) m_x-$Cl(f^{-1}(c_\mu(i_\mu(c_\mu(B))))) \subseteq f^{-1}(c_\mu(B))$ for every subset B of Y ;

(5) $f^{-1}(i_\mu(B)) \subseteq m_x$-$Int(f^{-1}(i_\mu(c_\mu(i_\mu(B)))))$ for every subset B of Y ;

(6) $f^{-1}(V)$ is m_x-open in X for every μ- regular open subset V of Y ;

(7) $f^{-1}(F)$ is m_x-closed in X for every μ- regular closed subset F of Y

Proof. (1) $\rightarrow$(2) Let V be a μ-open subset Y and $x \in f^{-1}(V)$. There exists a m_x-open subset U of X containing x such that $f(U) \subseteq i_\mu(c_\mu(V))$. This implies $x \in m_x$-$Int(f^{-1}(i_\mu(c_\mu(V))))$. Hence $f^{-1}(V) \subseteq m_x$-$Int(f^{-1}(i_\mu(c_\mu(V))))$.

(2) $\longrightarrow$(3) Let F be a μ-closed subset Y. Then $Y-F$ is μ-open subset Y. By (2), we have $f^{-1}(Y-F)\subseteq m_x$-$Int(f^{-1}(i_\mu(c_\mu(Y-F))))$

$=i_\mu(f^{-1}(Y-(c_\mu(i_\mu(F)))))$

$=X-(c_\mu(f^{-1}(c_\mu(i_\mu(F)))))$. Hence, m_x-$Cl(f^{-1}(c_\mu(i_\mu(F))))\subseteq f^{-1}(F)$.

(3) $\longrightarrow$(4) Let B be a subset of Y. Since $c_\mu(B)$ is μ-closed and by (3), m_x-$Cl(f^{-1}(c_\mu(i_\mu(c_\mu(B)))))\subseteq f^{-1}(c_\mu(B))$.

(4) $\longrightarrow$(5) Let B be a subset of Y. By (4)

$f^{-1}(i_\mu(B))=X-f^{-1}(c_\mu(Y-B))\subseteq X-(c_\mu(f^{-1}(c_\mu(i_\mu(c_\mu(Y-B))))))$

$=m_x$-$Int(f^{-1}(i_\mu(c_\mu(i_\mu(B)))))$

(5) $\longrightarrow$(6) Let V be any μ-regular open subset of Y. Since $i_\mu(c_\mu(i_\mu(V)))=V$. From (5) ,it follows $f^{-1}(V)\subseteq m_x$-$Int(f^{-1}(V))$ and so $f^{-1}(V)=m_x$-$Int(f^{-1}(V))$.

(6) $\longrightarrow$(7) Let F be any μ-regular closed subset of Y. Then by (6),

$X-f^{-1}(F)=f^{-1}(Y-F)=m_x$-$Int(f^{-1}(Y-F))=m_x$-$Int(X-f^{-1}(F))$.

Hence, $f^{-1}(F)=m_x$-$Cl(f^{-1}(F))$.

(7) $\longrightarrow$(1) Let V be any μ-regular open subset of Y containing $f(x)$. By (7) , $X-f^{-1}(V)=f^{-1}(Y-V)=m_x$-$Cl(f^{-1}(Y-V))=X-m_x$-$Int(f^{-1}(V))$. Since $x\in f^{-1}(V)=m_x$-$Int(f^{-1}(V))$, there exist a m_x-open set U containing x such that $U\subseteq f^{-1}(V)$.Hence by theorem 3.2.5 (4), f is almost (m,μ)-continuous.

Theorem 3.2.7 Let (X,m_x) be a minimal structure space and (Y,μ) be a generalized topological space. For a function $f:(X,m_x)\longrightarrow(Y,\mu)$, the following properties are equivalent:

(1) f is almost (m,μ)-continuous;

(2) m_x-$Cl(f^{-1}(U))\subseteq f^{-1}(c_\mu(U))$ for every μ-β-open subset U of Y;

(3) m_x-$Cl(f^{-1}(U))\subseteq f^{-1}(c_\mu(U))$ for every μ-semi open subset U of Y

(4) $f^{-1}(U)\subseteq m_x$-$Int(f^{-1}(i_\mu(c_\mu(U))))$ for every μ-preopen U of Y

Proof. (1) $\rightarrow$(2) Let U be any μ-β-open subset of Y. Since $c_\mu(U)$ is μ-regular closed. By Theorem 3.2.6 (7)m_x-$Cl(f^{-1}(c_\mu(U))) = f^{-1}(c_\mu(U))$. Thus m_x-$Cl(f^{-1}(U)) \subseteq m_x$-$Cl(f^{-1}(c_\mu(U))) = f^{-1}(c_\mu(U))$.

(2) $\rightarrow$(3) It is obvious since every μ-semi open set is μ-β-open.

(3) $\rightarrow$(1) Let F be any μ-regular closed subset of Y. Since F is μ-semi open, we have m_x-$Cl(f^{-1}(F)) \subseteq f^{-1}(c_\mu(F)) = f^{-1}(F)$. Thus from Theorem 3.2.6 (7), f is almost (m,μ)-continuous.

(1) $\rightarrow$(4) Let U be any μ-preopen subset of Y. Then $U \subseteq i_\mu(c_\mu(U))$ and $i_\mu(c_\mu(U))$ is μ-regular open. By Theorem3.2.6 (6), $f^{-1}(i_\mu(c_\mu(U)))$ $= m_x$-$Int(f^{-1}(i_\mu(c_\mu(U))))$. Thus, we have $f^{-1}(U) \subseteq f^{-1}(i_\mu(c_\mu(U)))$ $= m_x$-$Int(f^{-1}(i_\mu(c_\mu(U))))$.

(4) $\rightarrow$(1) Let U be any μ-regular open subset of Y. Thus U is μ-preopen and $f^{-1}(U) \subseteq m_x$-$Int(f^{-1}(i_\mu(c_\mu(U)))) = m_x$-$Int(f^{-1}(U))$. By Theorem 3.2.6 (6), f is almost(m,μ)-continuous.

CHAPTER 4

WEAKLY (m,μ)-CONTINUOUS FUNCTIONS

4.1 Weakly (m,μ)-Continuous Functions

In this section, we introduce and study weakly (m,μ)-continuous functions and investigate some of their characterizations.

Definition 4.1.1 Let (X,m_x) be a minimal structure space and (Y,μ) be a generalized topological space. A function $f:(X,m_x)\to(Y,\mu)$ is said to be weakly (m,μ)-continuous at a point $x\in X$ if for each μ-open set V containing $f(x)$, there exists a m_x-open set U containing x such that $f(U)\subseteq c_\mu(V)$. A function $f:(X,m_x)\to(Y,\mu)$ is said to be weakly (m,μ)-continuous if it has this property at each point $x\in X$.

Example 4.1.2 Let X={1, 2, 3}, m_x = {∅, {1}, {1, 2}, X}, Y ={a, b, c} and μ ={∅, {a}, {a, b}, Y}. Define $f:(X,m_x)\to(Y,\mu)$ as follows : f(1)=a, f(2)=b and f(3)=c. Then f is weakly (m,μ)-continuous.

Remark 4.1.3 From the definitions of (m,μ)-continuity, Almost (m,μ)-continuity and weakly (m,μ)-continuity, we have a following implications but the reverse relations may not be true in general:

$$(m,\mu)\text{-continuous} \Rightarrow \text{almost}(m,\mu)\text{-continuous} \Rightarrow \text{weakly}(m,\mu)\text{-continuous}$$

Example 4.1.4 Let X = {a, b}, m_x = {∅, {a}, X}, Y = {1, 2} and μ ={∅, Y}. Define $f:(X,m_x)\to(Y,\mu)$ as follows: f(a) = 1 and f(b) = 2.
Then f is weakly (m,μ)-continuous but it is not almost(m,μ)-continuous.

Theorem 4.1.5 Let (X,m_x) be a minimal structure space and (Y,μ) be a generalized topological space. A function $f:(X,m_x)\to(Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-continuous at $x\in X$;

(2) $x\in m_x\text{-}Int(f^{-1}(c_\mu(V)))$ for each μ-open set V containing $f(x)$.

Proof. (1) $\to$(2) Let $x \in X$ and V be a μ-open set containing $f(x)$. Then, there exists m_x-open set U containing x such that $f(U) \subseteq c_\mu(V)$. Then we have $x \in U \subseteq f^{-1}(c_\mu(V))$ and hence $x \in m_x\text{-}Int(f^{-1}(c_\mu(V)))$.

(2) $\to$(1) Let V be a μ-open set containing $f(x)$ such that $x \in m_x\text{-}Int(f^{-1}(c_\mu(V)))$. Put $U = m_x\text{-}Int(f^{-1}(c_\mu(V)))$. Then U is m_x-open containing x and $f(U) \subseteq c_\mu(V)$.

Theorem 4.1.6 Let (X, m_x) be a minimal structure space and (Y, μ) be a generalized topological space. A function $f : (X, m_x) \to (Y, \mu)$ is weakly (m, μ)- continuous if only if $f^{-1}(V) \subseteq m_x\text{-}Int(f^{-1}(c_\mu(V)))$ for every μ-open set V of Y.

Proof. $(\to)$ Let $x \in f^{-1}(V)$. Then $f(x) \in V$. Since f is weakly (m, μ)-continuous at x. By Theorem 4.1.5, we have $x \in m_x\text{-}Int(f^{-1}(c_\mu(V)))$ and hence $f(V) \subseteq m_x\text{-}Int(f^{-1}(c_\mu(V)))$.

$(\leftarrow)$ Let V be a μ-open set of Y containing $f(x)$. Then, we have $x \in f^{-1}(V) \subseteq m_x\text{-}Int(f^{-1}(c_\mu(V)))$. By Theorem 4.1.5, f is weakly (m, μ)-continuous.

4.2 Some Characterizations

In this section, we investigate the characterizations of weakly (m, μ)-continuous.

Theorem 4.2.1 For a function $f : (X, m_x) \to (Y, \mu)$, where m_x has property B , the following properties are equivalent:

(1) f is weakly (m, μ)-continuous;

(2) $f^{-1}(U) \subseteq m_x\text{-}Int(f^{-1}(c_\mu(U)))$ for every μ-open subset V of Y;

(3) $m_x\text{-}Cl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(F)$ for every μ-closed subset F of Y;

(4) $m_x\text{-}Cl(f^{-1}(i_\mu(c_\mu(A)))) \subseteq f^{-1}(c_\mu(A))$ for every subset A of Y;

(5) $f^{-1}(i_\mu(A)) \subseteq m_x\text{-}Int(f^{-1}(c_\mu(i_\mu(A))))$ for every subset A of Y;

(6) $m_x\text{-}Cl(f^{-1}(U)) \subseteq f^{-1}(c_\mu(U))$ for every μ-open subset U of Y.

Proof. (1) $\to$(2) Let U be any μ-open subset of Y and $x \in f^{-1}(U)$. There exists a m_x-open subset V of X containing x such that

$f(V)\subseteq c_{\mu}(U)$. Since $x\in V\subseteq f^{-1}(c_{\mu}(V))$, $x\in m_x$-$Int(f^{-1}(c_{\mu}(U)))$. Hence $f^{-1}(U)\subseteq m_x$-$Int(f^{-1}(c_{\mu}(U)))$.

(2) $\rightarrow$(3) Let F be any μ-closed subset of Y. Then $Y-F$ is μ-open subset of Y.By (2),

$f^{-1}(Y-F)\subseteq m_x$-$Int(f^{-1}(c_{\mu}(Y-F)))=m_x$-$Int(f^{-1}(Y-(i_{\mu}(F))))$

$=X-m_x$-$Cl(f^{-1}(i_{\mu}(F)))$. Thus, m_x-$Cl(f^{-1}(i_{\mu}(F)))\subseteq f^{-1}(F)$.

3) $\rightarrow$(4) Let A be a subset of Y. Since $c_{\mu}(A)$ is μ-closed in Y, from (3) , it follows m_x-$Cl(f^{-1}(i_{\mu}(c_{\mu}(A))))\subseteq f^{-1}(c_{\mu}(A))$.

(4) $\rightarrow$(5) Let A be a subset of Y. By (4), it follows

$f^{-1}(i_{\mu}(A))=X-f^{-1}(c_{\mu}(Y-A))\subseteq X-(m_x$-$Cl(f^{-1}(i_{\mu}(c_{\mu}(Y-A)))))$

$=m_x$-$Int(\ (f^{-1}(i_{\mu}(c_{\mu}(A))))$.

Thus, $f^{-1}(i_{\mu}(A))\subseteq m_x$-$Int(f^{-1}(c_{\mu}(i_{\mu}(A))))$.

(5) $\rightarrow$(6) Let U be any μ-open subset of Y. Suppose that $x\notin f^{-1}(c_{\mu}(U))$. Then $f(x)\notin c_{\mu}(U)$ and so there exists a μ-open subset of V containing $f(x)$ such that $U\cap V=\phi$ and so $c_{\mu}(V)\cap U=\phi$. By (5), it follows that $x\in f^{-1}(V)\subseteq m_x$-$Int(f^{-1}(c_{\mu}(V)))$. There exists a m_x-open set M containing x such that $x\in M\subseteq f^{-1}(c_{\mu}(V))$. Since $c_{\mu}(V)\cap U=\phi$ and $f(M)\subseteq c_{\mu}(V)$, we have $M\cap f^{-1}(U)=\phi$ and so $x\notin m_x$-$Cl(f^{-1}(U))$. Hence m_x-$Cl(f^{-1}(U))\subseteq f^{-1}(c_{\mu}(U))$.

(6) $\rightarrow$(1) Let $x\in X$ and U be any μ-open subset of Y containing $f(x)$.From $U=i_{\mu}(U)\subseteq i_{\mu}(c_{\mu}(U))$ and (6), $x\in f^{-1}(U)\subseteq f^{-1}(i_{\mu}(c_{\mu}(U)))$

$$
\begin{aligned}
&=X-f^{-1}(c_{\mu}(Y-(c_{\mu}(U))))\\
&\subseteq X-(m_x\text{-}Cl(f^{-1}(Y-(c_{\mu}(U)))))\\
&=m_x\text{-}Int(f^{-1}(c_{\mu}(U))).
\end{aligned}
$$

So there exists a μ-open subset V of X containing x such that $V\subseteq f^{-1}(c_{\mu}(U))$. Hence f is weakly (m,μ)-continuous.

Theorem 4.2.2 For a function $f:(X,m_x)\rightarrow(Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-continuous;

(2) m_x-$Cl(f^{-1}(i_{\mu}(F)))\subseteq f^{-1}(F)$ for every μ-regular closed subset F of Y;

(3) m_x-$Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-β-open subset G of Y;

(4) m_x-$Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-semi open subset G of Y.

Proof. (1) $\rightarrow$ (2) Let F be any μ-regular closed subset of Y. Then $i_\mu(F)$ is μ-open, by Theorem 4.2.1 (6), we have

m_x-$Cl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(c_\mu(i_\mu(F)))$.

Since F is μ-regular closed, we have m_x-$Cl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(c_\mu(i_\mu(F))) \subseteq f^{-1}(F)$.

(2) $\rightarrow$ (3) Let G be any μ-β-open set. Then $c_\mu(G) \subseteq c_\mu(i_\mu(c_\mu(G))) \subseteq c_\mu(G)$, so that $c_\mu(G)$ is μ-regular closed. From (2), it follows m_x-$Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$.

(3) $\rightarrow$ (4) Since every μ-semi open set is μ-β-open, it is obvious.

(4) $\rightarrow$ (1) Let U be any μ-open subset of Y. Then from (4), it follows m_x-$Cl(f^{-1}(U)) \subseteq m_x$-$Cl(f^{-1}(i_\mu(c_\mu(U)))) \subseteq f^{-1}(c_\mu(U))$. Hence, by Theorem 4.2.1 (6), f is weakly (m,μ)-continuous.

Theorem 4.2.3 For a function $f:(X,m_x) \rightarrow (Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-continuous;

(2) m_x-$Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-preopen subset G of Y;

(3) m_x-$Cl(f^{-1}(G)) \subseteq f^{-1}(c_\mu(G))$ for every μ-preopen subset G of Y;

(4) $f^{-1}(G) \subseteq m_x$-$Int(f^{-1}(c_\mu(G)))$ for every μ-preopen subset G of Y.

Proof. (1) $\rightarrow$ (2) Let G be any μ-preopen subset of Y. Then $c_\mu(G) = c_\mu(i_\mu(c_\mu(G)))$, so $c_\mu(G)$ is μ-regular closed. From Theorem 4.2.2 (2), it follows that m_x-$Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$.

(2) $\rightarrow$ (3) Let G be any μ-preopen subset of Y. Then $G \subseteq i_\mu(c_\mu(G))$ and by (2), we have m_x-$Cl(f^{-1}(G)) \subseteq m_x$-$Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$.

(3) $\rightarrow$ (4) Let G be any μ-preopen subset of Y. By (3), it follows that $f^{-1}(G) \subseteq f^{-1}(i_\mu(c_\mu(G))) = X - f^{-1}(c_\mu(Y-(c_\mu(G)))) = X - m_x$-$Cl(f^{-1}(Y-(c_\mu(G))))$

$= m_x\text{-}Int(f^{-1}(c_\mu(G)))$. Hence, $f^{-1}(G) \subseteq m_x\text{-}Int(f^{-1}(c_\mu(G)))$.

(4) $\rightarrow$ (1) Since every μ-open set is μ-preopen, from (4), Theorem 4.2.1 (2), it follows f is weakly (m,μ)-continuous.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATION

5.1 Conclusions

This thesis has been concerned with the (m,μ)-continuous functions on minimal structure space . First, we introduced the concepts of the functions, $f:(X,m_x)\rightarrow(Y,\mu)$ is said to be (m,μ)- continuous at a point $x\in X$ if for each μ-open set V containing $f(x)$, there exists a m_x-open set U containing x such that $f(U)\subseteq V$. A function $f:(X,m_x)\rightarrow(Y,\mu)$ is said to be (m,μ)-continuous if it has this property at each point $x\in X$. Next, we defined a function $f:(X,m_x)\rightarrow(Y,\mu)$ is said to be almost (m,μ)-continuous functions at a point $x\in X$ if for each μ-open set V containing $f(x)$, there exists a m_x-open set U containing x such that $f(U)\subseteq i_\mu(c_\mu(V))$. A function $f:(X,m_x)\rightarrow(Y,\mu)$ is said to be almost (m,μ)- continuous if it has this property at each point $x\in X$ and more attractive properties as follow:

1. For a function $f:(X,m_x)\rightarrow(Y,\mu)$, the following properties are equivalent:

(1) f is (m,μ)-continuous at $x\in X$;

(2) $x\in m_x$- $Int(f^{-1}(V))$ for every $V\in\mu$ containing $f(x)$;

(3) $x\in f^{-1}(c_\mu(f(A)))$ for every subset A of X with $x\in m_x$- $Cl(A)$;

(4) $x\in f^{-1}(c_\mu(B))$ for every subset B of Y with $x\in m_x$- $Cl(f^{-1}(B))$;

(5) $x\in m_x$- $Int(f^{-1}(B))$ for every subset B of Y with $x\in f^{-1}(i_\mu(B))$;

(6) $x\in f^{-1}(F)$ for every μ- closed set F of Y such that $x\in m_x$-$Cl(f^{-1}(F))$.

2. For a function $f:(X,m_x)\rightarrow(Y,\mu)$, the following properties are equivalent:

(1) f is almost (m,μ)-continuous at $x\in X$;

(2) $x\in m_x$-$Int(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open set V containing $f(x)$;

(3) $x\in m_x$-$Int(f^{-1}(V))$ for every μ- regular open set V containing $f(x)$;

(4) For every μ- regular open set V containing $f(x)$, there exists a μ-open set U containing x such that $f(U)\subseteq V$.

3. For a function $f:(X,m_x)\to(Y,\mu)$ which m_x has property B , the following properties are equivalent:

(1) f is almost (m,μ)-continuous;

(2) $f^{-1}(V)\subseteq m_x$ -$Int(f^{-1}(i_\mu(c_\mu(V))))$ for every μ-open subset V of Y;

(3) m_x-$Cl(f^{-1}(i_\mu(F)))\subseteq f^{-1}(F)$ for every μ-closed subset F of Y;

(4) m_x-$Cl(f^{-1}(c_\mu(i_\mu(c_\mu(B)))))\subseteq f^{-1}(c_\mu(B))$ for every subset B of Y;

(5) $f^{-1}(i_\mu(B))\subseteq m_x$-$Int(f^{-1}(i_\mu(c_\mu(i_\mu(B)))))$ for every subset B of Y;

(6) $f^{-1}(V)$ is m_x-open in X for every μ-regular open subset V of Y;

(7) $f^{-1}(F)$ is m_x-closed in X for every μ-regular closed subset F of Y.

4. For a function $f:(X,m_x)\to(Y,\mu)$, the following properties are equivalent:

(1) f is almost (m,μ)-continuous;

(2) m_x-$Cl(f^{-1}(U))\subseteq f^{-1}(c_\mu(U))$ for every μ-β-open subset U of Y;

(3) m_x-$Cl(f^{-1}(U))\subseteq f^{-1}(c_\mu(U))$ for every μ-semi open subset U of Y;

(4) $f^{-1}(U)\subseteq m_x$-$Int(f^{-1}(i_\mu(c_\mu(U))))$ for every μ-preopen U of Y.

And last, we introduced a function $f:(X,m_x)\to(Y,\mu)$ is said to be weakly (m,μ)-continuous at a point $x\in X$ if for each μ-open set V containing $f(x)$, there exists a m_x- open set U containing x such that $f(U)\subseteq c_\mu(V)$. A function $f:(X,m_x)\to(Y,\mu)$ is said to be weakly (m,μ)-continuous if it has this property at each point $x\in X$. From the definition of the function above, we have properties as follow:

1. For a function $f:(X,m_x)\to(Y,\mu)$, the following properties are equivalent:

(1) f is *weakly* (m,μ)-continuous at $x\in X$;

(2) $x\in m_x$-$Int(f^{-1}(c_\mu(V)))$ for each μ-open set V containing $f(x)$

2. For a function $f:(X,m_x)\to(Y,\mu)$ is weakly (m,μ)-continuous if only if $f^{-1}(V)\subseteq m_x$-$Int(f^{-1}(c_\mu(V)))$ for every μ-open set V of Y.

3. For a function $f:(X,m_x)\to(Y,\mu)$, where m_x has property B , the following properties are equivalent:

(1) f is weakly (m,μ)-continuous;

(2) $f^{-1}(U)\subseteq m_x$-$Int(f^{-1}(c_\mu(U)))$ for every μ-open subset V of Y;

(3) m_x-$Cl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(F)$ for every μ-closed subset F of Y

;

(4) m_x-$Cl(f^{-1}(i_\mu(c_\mu(A)))) \subseteq f^{-1}(c_\mu(A))$ for every subset A of Y;

(5) $f^{-1}(i_\mu(A)) \subseteq m_x$-$Int(f^{-1}(c_\mu(i_\mu(A))))$ for every subset A of Y;

(6) m_x- $Cl(f^{-1}(U)) \subseteq f^{-1}(c_\mu(U))$ for every μ-open subset U of Y.

4. For a function $f:(X,m_x) \rightarrow (Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-continuous;

(2) m_x-$Cl(f^{-1}(i_\mu(F))) \subseteq f^{-1}(F)$ for every μ - regular closed subset F of Y;

(3) m_x-$Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-β- open subset G of Y;

(4) m_x-$Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ-semi open subset G of Y.

5. For a function $f:(X,m_x) \rightarrow (Y,\mu)$, the following properties are equivalent:

(1) f is weakly (m,μ)-continuous;

(2) m_x-$Cl(f^{-1}(i_\mu(c_\mu(G)))) \subseteq f^{-1}(c_\mu(G))$ for every μ- preopen subset G of Y;

(3) m_x-$Cl(f^{-1}(G)) \subseteq f^{-1}(c_\mu(G))$ for every μ- preopen subset G of Y;

(4) $f^{-1}(G) \subseteq m_x$-$Int(f^{-1}(c_\mu(G)))$ for every μ- preopen subset G of Y.

I have finally discovered that certain relationship of (m,μ)-continuous functions, almost (m,μ)- continuous functions and weakly (m,μ)-continuous functions as follows that (m,μ)- continuous functions always imply almost (m,μ)- continuous functions and almost (m,μ)- continuous functions, we have a following (m,μ)- continuous functions implication but the reverse relation may not be true in general:

(m,μ)-continuous $\Rightarrow$ almost (m,μ)-continuous $\Rightarrow$ weakly (m,μ)- continuous

5.2 Recommendations

To this end, even though I gave found several properties as presented in this thesis, there are several questions yet to be answered and it may be worth investigating in future studies. I formulate the questions as follows :

1. Are there any properties of (m,μ)-continuous functions, almost (m,μ)-continuous functions and weakly (m,μ)-continuous functions?
2. Is there any property of these functions on other space?
3. Do these functions have any connections with others?

REFERENCES

REFERENCES

[1] Csaszar, *A. Generalized topology, generalized continuity*, Acta Math. Hungar.,96 (2002), 351–357.

[2] Csaszar, *A. Generalized open sets in generalized topologies*, Acta Math. Hungar., 106 (2005), 53–66.

[3] Boonpok, C., *Almost (g,m)-Continuous Function*, Int. Math. Forum, 2010 ; 5(40) : 1957 – 1964.

[4] Boonpok, C., *Weakly (g,m) - Continuous Function*, Int. Journal of Math. Analysis, 2010 ; 4(40) : 1965 – 1673.

[5] Boonpok, C., *Biminimal structure space*, International Mathematic Forum, 2010; 15(5) : 703 – 707.

[6] Min,K. M. *A note on $\theta(g, g')$-continuity in generalized topological space*, Acta Math. Hungar, 125 (2009), 387–393.

[7] Noiri T. and Popa V., *A generalized of some of g-irresolute functions*, J. of Pure and Appl. Math. 4(4)(2009), 473 – 493.

BIOGRAPHY

BIOGRAPHY

Name	Miss Supawimol Pholdee
Date of birth	March 20, 1986
Place of birth	Yasothon Province, Thailand
Instisution Attended	
2004	Mattayom 6 in Yasothonpittayakom School in Yasothon , Thailand.
2009	Bachelor of Education in Mathematics Education (first honor)from Khonkaen University in
2012	Thailand. And Master of Science in Mathematics Education, Mahasarakham Universiy in Thailand.
Contact address	40/1 Jaengsanit 2 Soi, Tambon Naimuang, Muang District Yasothon Province, 35000. hemogobin_A@hotmail.com hemogobin_AA@yahoo.com